THE SPRING MANUFACTURERS' RESEARCH ASSOCIATION

# Laminated Torsion Bars
# A Literature Survey

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by

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June 1966

THE SPRING MANUFACTURERS' RESEARCH ASSOCIATION

# Laminated Torsion Bars - A Literature Survey

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under Contract No. KS/2A/0218/CB.43(b)2

## Summary

This literature survey has shown that there is very little published information on the characteristics of laminated torsion bars. Several advantages are claimed for this type of spring over conventional torsion bars, but only empirical design information has been found for the stiffness of or stresses in a laminated torsion bar. The design information has been found to be restricted to torsion bars of specific dimensions and with end fittings which allow sideways movement of the leaves in the end chucks. This survey has been carried out on behalf of the Ministry of Aviation, Directorate of Guided Weapons Research, prior to undertaking a research investigation to provide a reliable basis for the design of springs of this type.

(June, 1966)

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CONTENTS

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## 1. INTRODUCTION

A laminated torsion bar consists essentially of a pack of rectangular leaves held together only at the points of torque application. Thus each leaf can move independently of its neighbour at all but the ends of the pack. Fig. (1) illustrates such a bar. When torque is applied, the outer leaves move over the inner ones giving a lower spring rate than that of a solid bar with the same length and cross section.

The first published article on laminated torsion bars appeared in 1949[1]. This described development work being carried out to investigate the design of a road vehicle suspension unit incorporating such a spring.

In 1951, it was reported[2] that a design for a car suspension unit made up of a pack of 7 rectangular leaves working in torsion had been produced and tested.

Another early use of these springs was on a continental vehicle front suspension unit. Reports suggested that the suspension was designed at a time when round solid torsion bar material was not as readily available as flat spring steel, which was also cheaper. The laminated bars were probably designed on a trial and error basis with only the one application in view.

Prior to this, interest in torsion bars of thin rectangular section had grown in the U.S.A. and a U.S. patent by J.O. Almen[3] was filed on the varying rate characteristics of pretwisted bars of this shape. Later work included the filing of another U.S. Patent in 1950[4] by P.Z. Anderson, which described the invention of a socket to hold bars of square or rectangular cross section subjected to torsion. This was particularly applicable to springs of the type suggested by J.O. Almen, and H.O. Fuchs[5] suggested that this type of socket should be used for gripping flat bars. A number of patents[6][7][8][9][10][11][12] other than those by Almen and Anderson have been filed on specific aspects of the design and application of laminated torsion bars, but no reported experimental evidence has been found to support the advantages claimed by the inventors.

Published material on laminated torsion bar development is limited. The only published reports directly applicable to the subject, apart from those[1][2] already referred to, are one by R.E. Hanslip and L.O. Imber[13] which discussed future research as well as design, and one by S. Mordasewicz[14] which referred to torsion bars as "Fasciculated Springs" and suggested design formulae for torque and shear stress.

## 2. FEATURES OF THE DESIGN

When research workers have considered the advantages of laminated torsion bars, they have usually compared them with solid torsion bars rather than coil or leaf springs. It is, however, worth noting that the laminated torsion bar is a spring for which both significant damping and variable rate are claimed. In his report[14] S. Mordasewicz contended that the solid torsion bar was the only logical choice of spring element, pointing out that the energy stored in a given space and for a given weight was much higher with these bars than with any other kind of spring. He extended this logic to laminated torsion bars, pointing out that these were much shorter than a single solid bar for the same rate, even though the energy/lb was slightly lower.

The main features of these springs and the main points raised in the literature comparing these torsion bars with other spring types are individually discussed, but all the articles on laminated bars suggest that an advantage is to be gained by the fact that if one bar fails, the spring can still function until a replacement is found.

(a) Short length for a given rate and maximum stress

When a solid round torsion bar is designed for a specific application, the length and diameter are fixed by the rate and maximum stress desired. With a laminated torsion bar, there are more factors involved in the design, namely the number, width and thickness of leaves, which may be varied to give a desired length, rate and maximum stress, and the nature of the fixing. This is probably the most obvious advantage to be gained by their use in vehicle applications where round bars have to be very long to give the desired rate. It was shown by H.O. Fuchs(5), in his unpublished report that for a single rectangular bar, compared with a solid round torsion bar of similar stress and flexibility, the ratio of the lengths with different width to thickness ratios of the rectangular bar was as shown in Fig. (2). In this, curve "A" shows the comparison with neither bar preset, and curve "B" with both bars preset. It may be surprising to see that even for a single rectangular bar the active length can be much shorter than that of the round bar when the width to thickness ratio is high, i.e. when $\frac{t}{b}$ in Fig. (2) is small. This fact is particularly true for the bar when preset, curve "B", calculations for which were based on the sand heap analogy(15) for both bars. These graphs indicate the results obtained when using only one bar. With a laminated pack of bars in use, the results would tend to show off the length advantage of that design to a greater degree.

(b) Variable rate effects

When a single rectangular bar is twisted, the outer edge must form a helix the length of which is longer than that of its axis. The resulting longitudinal stresses and their effect on the torque are explained by S.Timoshenko(16) who gives the torque for the condition when the bar is twisted away from the flat by:-

$$T = \frac{bt^3G\theta}{3}\left(1 + \frac{1}{120}\,\frac{E}{G}\,\frac{b^4}{t^2}\,\theta^2\right)$$

and the maximum longitudinal stress produced in the outer edge of the plate as:-

$$f_t = \frac{E\theta^2 b^2}{12}$$

where $T$ = applied torque

$b$ = breadth of plate

$t$ = thickness of plate

$\theta$ = angle of twist per unit length (radians/unit length)

$f_t$ = maximum longitudinal tensile stress

$E$ = Young's Modulus of Elasticity

$G$ = Rigidity Modulus

In this derivation it is assumed that the rectangle is thin enough for the torque due to shearing stresses to be found from:-

$$T_s = \frac{bt^3 G\theta}{3} \quad \text{where } T_s = \text{torque due to shear only}$$

The resulting effect on the torque is seen from this to be due to $\theta^3$ which increases the rate at high values of twist, $\theta$. Shown in Fig. (3) is a comparison of this theory with experimental results for a single leaf given in an unpublished report to the Ministry of Supply[17].

The realisation that when a single thin rectangular bar was pretwisted and loaded towards the flat condition, a sinusoidal torque-deflection curve could be obtained, was patented by J.O. Almen[3]. This patent especially covers springs having zero and negative rates over a portion of the curve. Although only showing positive rate, Fig. (4) illustrates the principle involved. This figure is taken from design calculations by M. Olley[18] and is based on bar dimensions of 5.0 in x 0.237 in x 17.9 in long. The longitudinal stress effect is shown and superimposed upon the straight line graph which is drawn by calculating the torque produced by shear stress alone; the combination of these gives the varying rate effect shown. Almen said that the longitudinal stresses could be large enough to produce zero or negative rates over a limited range of twist when the bar was flat or near to flat, and the angle of pretwist ($\alpha_o$) needed for zero rate was given by:

$$\alpha_o = \sqrt{3}\ \frac{lt}{w^2} \simeq \frac{7\ lt}{b^2}$$

where $l$ = length of bar
$\alpha_o$ = angle of twist (radians)
$w$ = $\frac{1}{2}$ width of plate
$t$ = thickness of plate
$b$ = width of plate

It is suggested in this patent that more than one leaf could be used for the design of such a spring and it does seem that with laminated bars one could produce various effects by using a bar made with an initial twist such that the combined effect gives varying rates over the desired range of twist.

For single leaf bars, the zero or negative rates are limited by the maximum permissible stress which can limit the total windup angle available, the initial manufactured twist is then also limited since the bar only gives the zero rate when twisted flat at which point the maximum shear stress may be exceeded. In order to get these varying effects, the thickness to width ratio must, therefore, be high and it is suggested by R.E. Hanslip and L.O. Imber(13) that the ratio needed for zero or negative rate effects makes the design impractical and that the important practical use is for springs of increasing positive rates. There are suggestions made in the same report that if the bar is made with an initial twist and then preset towards the flat state, such that at the static working loads the bar will be near straight and compressive stresses are present in the edges, then the fatigue life will be improved. This is a further advantage to be gained by using a bar manufactured with initial twist.

(c) Damping

Although all the published papers on torsion bars of this type do mention interleaf friction, only two(1)(2) claim a definite amount of damping. This is substantiated by tests and the figure claimed is about 300% more than the equivalent solid torsion bar or coil spring. Hanslip and Imber say that the expected friction hysteresis loop does not appear in practice and according to one of their customers the effects of damping are negligible. No further written information is available on the subject, but verbal information resulting from work done for M.O.A.(GW(E)) indicates that significant damping can be demonstrated during dynamic loading.

(d) End fixings

One of the main difficulties experienced with round torsion bars is their end location. The only satisfactory methods tend to be costly and require that the ends of the bars be upset and machined. It is pointed out in some reports(1)(2) that if a laminated torsion bar, of rectangular section, is fitted into a rectangular hole for location then, since the fixing stresses are greatest at the position of least shear stress, a good location is easily provided. It seems, however, that the laminations should be made to fairly close tolerances on their thickness or else the overall pack size will vary too much for a good fit in the socket.

It has been suggested by Hanslip and Imber that a suitable end fixing may be formed by bending the end of the plate at right angles, as shown in Figs. (5) and (6).

Fuchs said that the fatigue failures invariably started near the plane of torque application and analysed the stresses for the case of a single flat bar with the torque applied by a rectangular socket. The reason given for these failures was that, for the portion of the bar in the socket, the narrow edge was parallel with the axis of the bar, whereas in the section of bar under twist it formed a helix (Fig. 7). At the edge of the socket, the change in direction was accompanied by stresses equivalent to bending stresses tensile on the outside of the curve at point "A" and compressive on the inside at point "B", the value of the tensile stress being approximately 127% of the maximum shear stress in the bar. The solution suggested in the report avoided this difficulty by using a "V" notch location, as patented by Anderson, which, besides permitting higher stresses, reduced the overall length of the spring as there was no inactive material for end fixing (Fig. 8). This patent can be applied to either single leaves or packs of leaves. A suggestion by Hanslip and Imber to have the two outside leaves of a laminated torsion bar thinner than the rest does seem to afford a possible solution if the use of a simple socket is to be preferred. Other types of end fittings have been patented[8][9][10][11][12], but no published reports are available to show the advantages of these end fittings.

(e) Weight

As the stresses in a rectangular leaf are not uniform across the section, the weight of a laminated torsion bar tends to be higher than that of a round torsion bar with the same maximum stress. This is regarded as the main disadvantage by Hanslip but Fuchs found that, allowing 5% advantage gained by using "V" slot end fixings, the weight ratio of round bar to flat bar of width to thickness ratio 10:1 is 7:10, the extra cost of which can be more than offset by the simplicity of manufacture. If the weight considerations are of great importance then both types of bar will invariably be prestressed and, because of the improvement in stress distribution the weight ratio becomes 8.5:10 without allowing for the extra saving available by using "V" slot location.

(f) Cost

Cheapness is probably the most important advantage claimed for the laminated torsion bar when compared with a solid round bar. If a standard size of leaf spring material could be used, then Hanslip pointed out that there was an immediate cost advantage to be gained. He also said that the manufacturing procedure could, if presetting was not required, consist of only cutting the bar to length, followed by heat treatment, thus making the total cost much cheaper than that of a solid round bar. It is also suggested that other cost advantages are to be gained by the simple end fixing and the fact that some surface seams and defects may not be as important as on round bars, since a leaf may split longitudinally without noticeably reducing the torque produced by the pack. The first report[1] suggested the possibility of using rolled carbon steel strip, which would be cheaper than the silicon-manganese or chromium vanadium steels then being investigated. Fuchs said that whilst the weight of the laminated torsion bar was necessarily higher than that of a solid round bar, the reduction in cost due to the simplicity of manufacture and design more than offset the increase in cost due to higher weight, so that the cost per unit of energy would be lower.

## 3. DESIGN FORMULAE FOR LAMINATED TORSION BARS

It has been suggested[13] that the torque developed by laminated torsion bars is substantially that of a single bar of the pack multiplied by the number of bars in the pack. This is probably a fair approximation for packs of few bars, but may not be sufficiently accurate for bars which are made up of many laminations. The following are two design formulae available for larger packs of leaves, but both appear to be limited in their application.

### (a) Unpublished report to Ministry of Supply[17]

The formulae for torque and stress were obtained on unspecified material and the leaf sizes tested were within the range 0.5 in to 1.0 in width, 0.018 in to 0.040 in thickness, up to 36 in number and from 6.75 in to 12 in active length. The formula given for stress was said to be a tentative one and both formulae are only applicable to bars whose ends are held in a manner which allows the leaves to pivot sideways in the socket whilst held together in a perpendicular plane. The torque (lbf in) was given by:-

$$T = \frac{4\,340\,000\,\alpha^{(.017\frac{b}{t} + .983)}\,(n + .008n^2)\;t^3 b^{1.44}}{L^{1.44}}$$

The maximum shear stress ($lbf/in^2$) was given by:-

$$S = 1.21\,\frac{T}{n}\left[\frac{3b + 2t}{b^2t^2}\right]$$

Where
- $\alpha$ = angle of twist taking $30^\circ$ as a unit
- $n$ = number of leaves
- $b$ = breadth of leaves (in)
- $t$ = thickness of leaves (in)
- $L$ = length under twist (in)

### (b) S. Mordasewicz[14]

The formulae for torque and stress apply to laminated torsion bars under the following conditions:-

- (i) All leaves have the same cross section
- (ii) The cross-section of the spring is square
- (iii) The number of leaves is equal to the width/thickness ratio (from (i) and (ii))
- (iv) The end sockets embody an adequate amount of side and end float

(v) All contact surfaces are ground, machine polished and constantly lubricated.

(vi) Angular prestressing and the use of shims, liners and inserts between the leaves are disregarded.

The formulae, although stated to be theoretical, are really empirical because they are based on previous designs of spring and experimental data for up to 22 leaves. The torque (lbf in) is given by:-

$$T = \frac{\theta G b^4}{L} (.471m^2 - .73m + .40\sqrt{m})$$

The maximum shear stress (lbf/in$^2$) is given by:-

$$S = \frac{\theta G b}{L} \left(\frac{3m + 1.8}{m^3}\right) (.625m^2 - 1.384m + 90\sqrt{m})$$

The only defined symbol in the article is "m", which is the width-thickness ratio; it does, on inspection, seem that b is the thickness of the leaves, θ is the angle of twist, L is length under twist and G the modulus of rigidity and the probable units used lbs, ins and radians.

Comparing the two analyses(17)(14) the end fixing used in these investigations allowed either pivoting or side float of the leaves in the socket and it is surprising to see such a difference in the torque and stress formulae. The numerical difference can be illustrated by calculating the torque for 22 leaves 0.66 in x 0.030 in x 10 in long, if the angle of twist is 90° the first formula gives a torque of 269 lb in, whereas the second gives a torque of 313 lb in.

A further report(19) gives a considerable number of test results, for springs with up to 61 leaves, 0.020 in to 0.042 in in thickness, 0.5 in to 1.25 in in width and 11.2 in to 13.7 in long, with end fixings which allowed sideways movement of the leaves. The formula suggested in (17), should apply to these test results and a comparison of theory and experiment is shown in Figs. (9) to (14). Also shown in these figures is the torque of one leaf, as calculated by the Timoshenko formula, multiplied by the number of leaves in the pack. It can be seen that either calculation gives a good approximation to the experimental results.

## 4. PRESENT APPLICATIONS

Probably one of the best known applications of laminated torsion bars is in the front suspension of a car, the general layout of which is shown in Fig. (15). The two laminated bars are inside the cross tubes and are located by a square internal section and a set screw at the centre of each tube. The link arms are similarly fitted on the end of each bar, being again located with a square internal section and a pin. This, in effect, gives each wheel two torsion bars which are half the width of the car. The bar is made up from chromium-vanadium spring steel which, according to a report by Scott[20] has been annealed for four hours at 450°C. This would appear to be an error and a hardening and tempering treatment is suggested by other sources. The bar used is made up of eight leaves, four being full width and four being half width strips, thus making a 0.72 in square pack of 6 layers, 37.5 in overall length as shown in Fig. (15). Scott[20] described a pack of bars welded together at the ends but the latest designs on the same vehicle avoid welding and use the bar without any interleaf connections. Scott stated that gradual weakening and occasional partial fracture had been experienced with these laminated bars, but manufacturing costs are low and replacements cheap. This car's design stresses reported by Hanslip[13] were 75 000 lbf/in$^2$ static and the trailer suspension mentioned in the same article and shown in Fig. (4) was said to have had a good fatigue life when operating from 75 000 to 125 000 lbf/in$^2$ without presetting or shot peening.

A car is at present using laminated torsion bars in the form of a 5 leaf pack made from material 1.410 in x 0.282 in x 31.3/8 in effective length, each individually hardened in a jig and tempered to a hardness of approximately 450 HV. In this design, the static shear stress is 75 000 lbf/in$^2$ and the maximum shear stress 100 000 lbf/in$^2$. It seems that the allowable stresses depend on the type of end fixing used, both the designs described use the square socket type of location, whereas in the trailer, the springs are bent over so that the trailing arms are integral with the spring.

## 5. CONCLUSIONS

In the field of laminated torsion bars, so little research has been published that considerable work is necessary on both the theoretical and practical aspects of design. A suggested programme in the report by Hanslip lists fourteen points, each needing individual investigation and, although this may appear to be fairly comprehensive, it is suggested in the same report that there are many more avenues to investigate. It seems, therefore, that primary work should be on the relationship between torque and twist and the determination of the permissible working stresses in the leaves for a wide range of torsion bar dimensions. These characteristics will almost certainly depend on the type of end fixing used, which should therefore be graded if possible into two or three general types for which the characteristics can be established.

The advantages claimed for laminated torsion bars indicate that there could be many more applications than at present if more design data were available. At present, if a designer wishes to use such springs, he must build and test several prototypes until a suitable one is developed for the purpose in mind. More general data would reduce empiricism.

## 6. REFERENCES


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(6) British patent 610,669; J.K. Bache.

(7) British patent 660,443; J.K. Bache.

(8) British patent 673,013; J.K. Bache, and A.A.Digby-Brown.

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(10) Canadian patent 712,488; J.G. Holmstrom.

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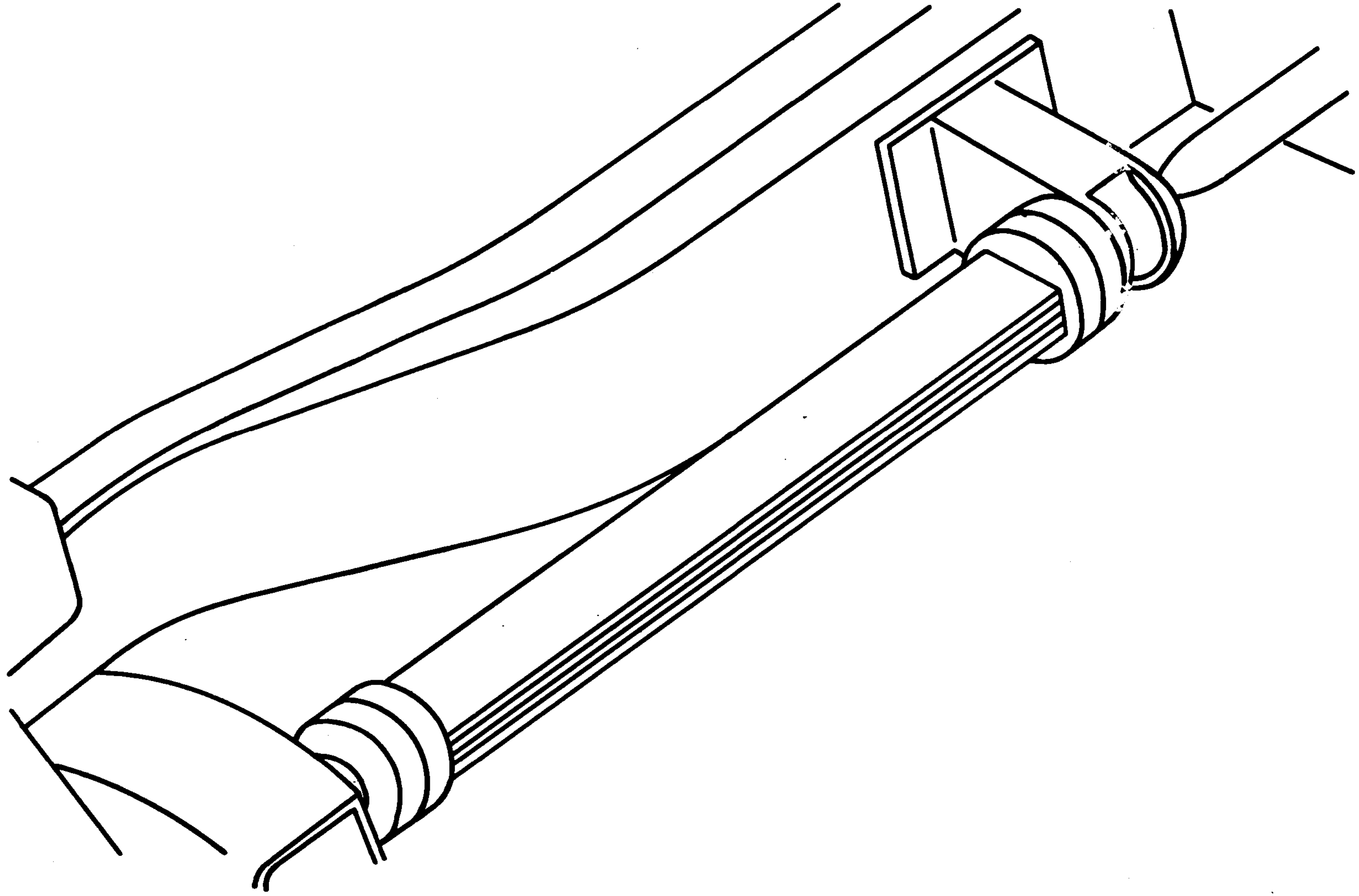


FIG. I. LAMINATED TORSION BAR USED IN VEHICLE FRONT SUSPENSION SYSTEM.

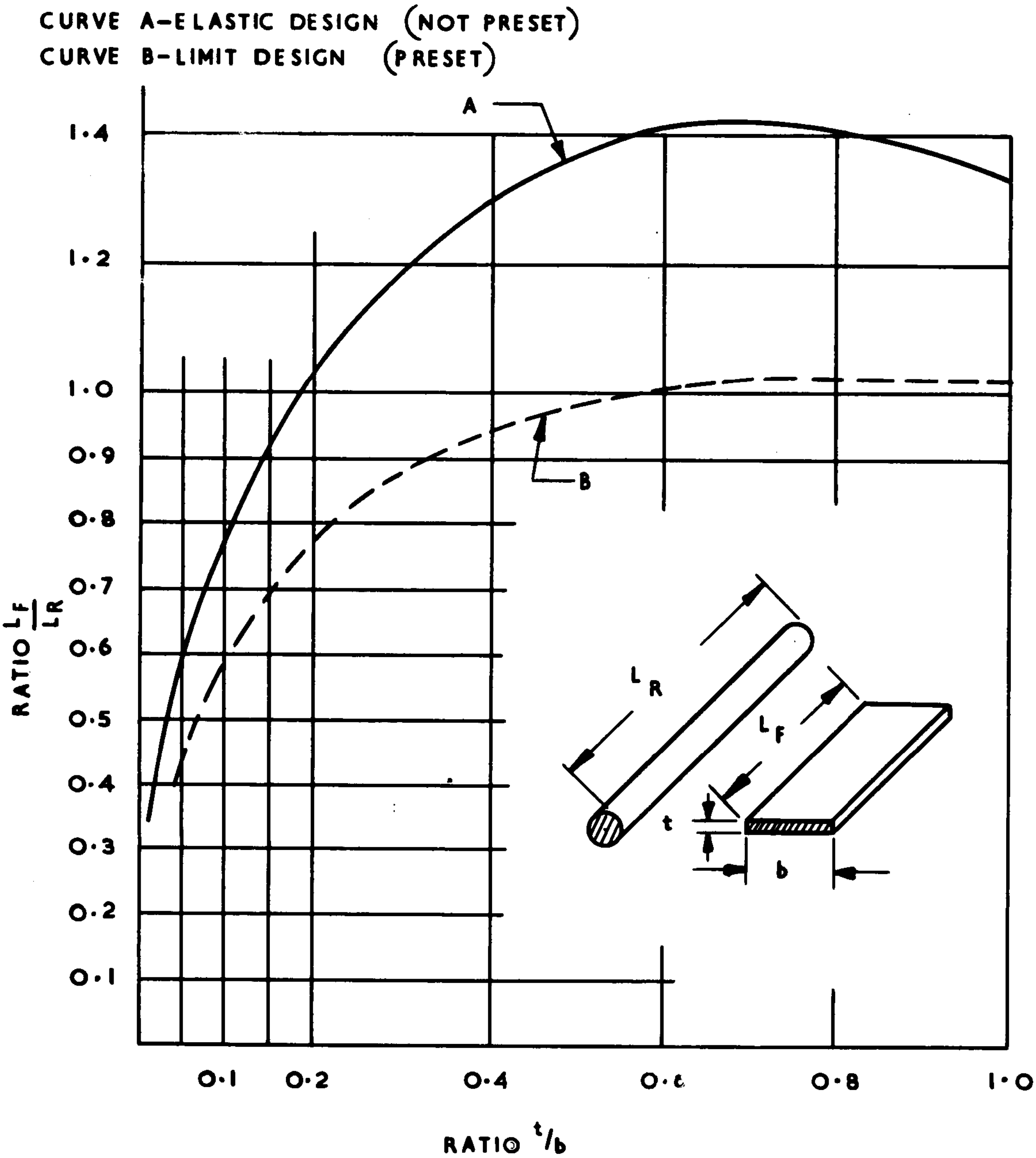



FIG. 2. COMPARISON OF LENGTH OF FLAT AND ROUND TORSION BARS OF SAME RATE, LOAD CAPACITY AND STRESS.

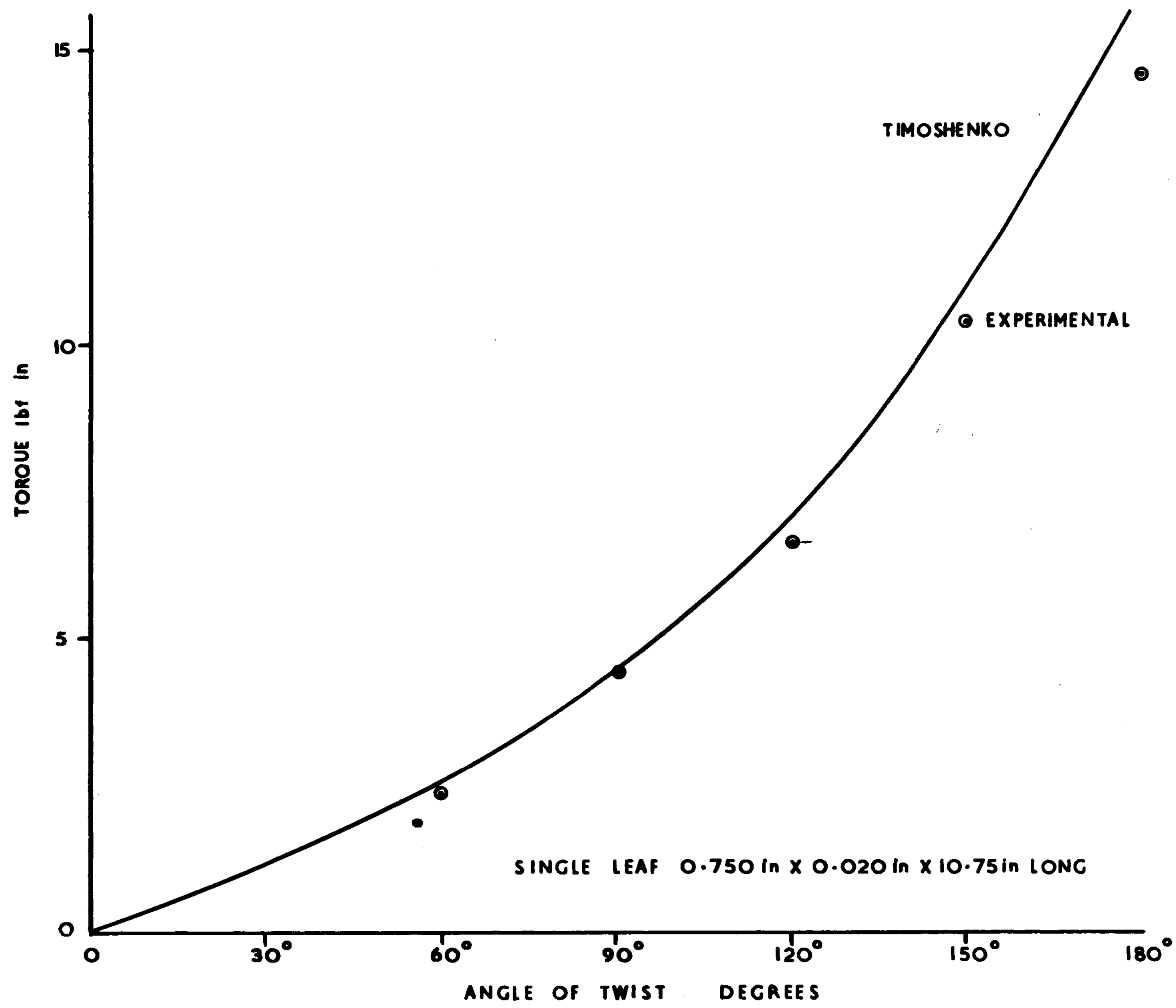



FIG. 3. COMPARISON OF TIMOSHENKO FORMULA WITH EXPERIMENTAL RESULTS

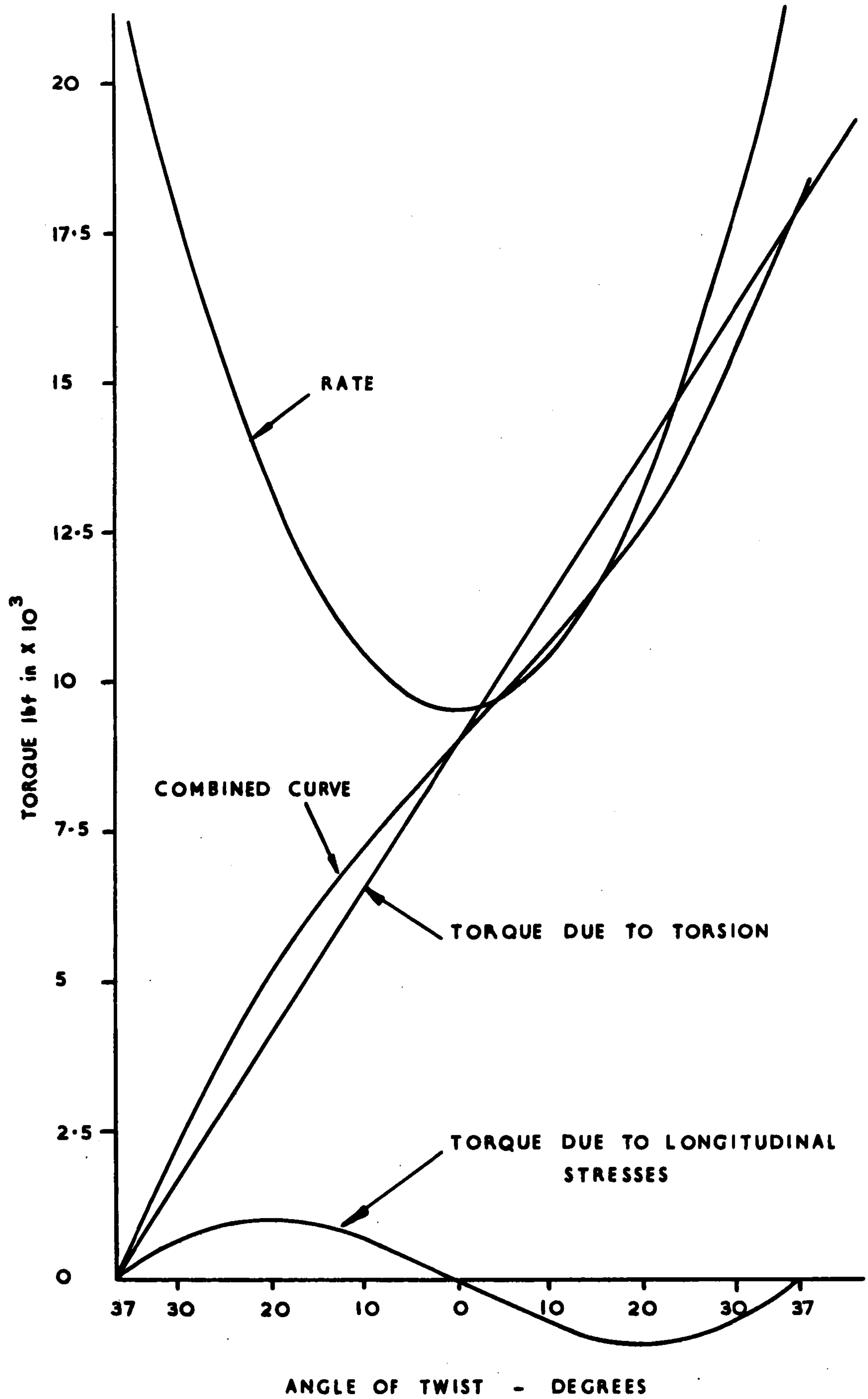



FIG. 4. SINGLE THIN RECTANGULAR BAR IN TORSION

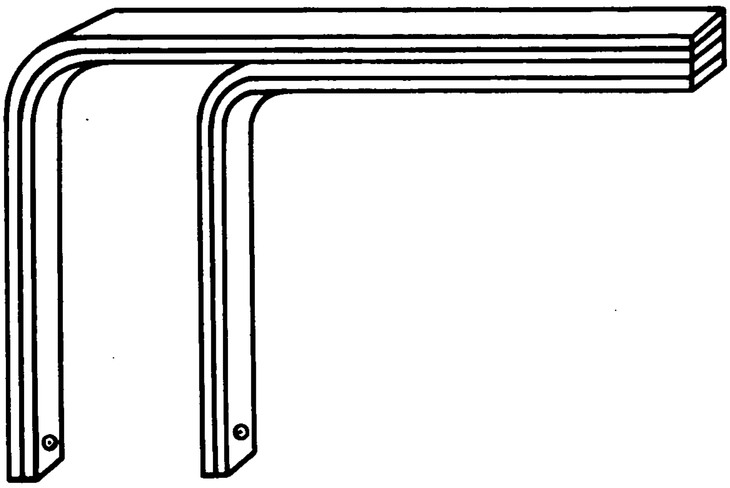


FIG. 5. TORSION BAR WITH INTEGRAL TRAILING LINKS AS USED IN BOAT TRAILER SUSPENSION.

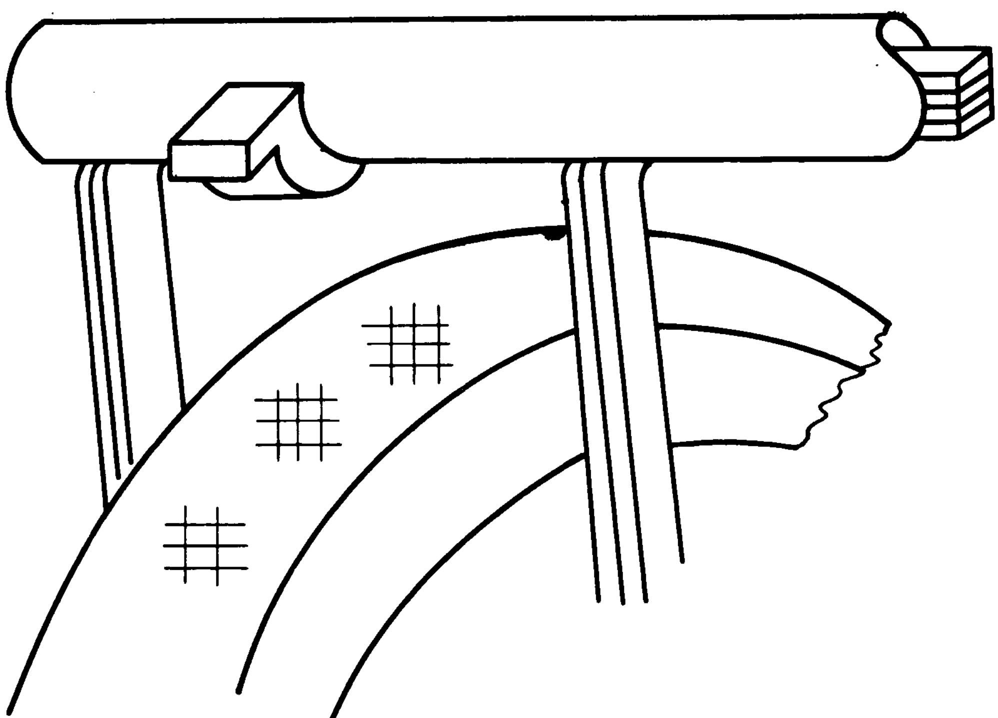


FIG. 6. BOAT TRAILER SUSPENSION.

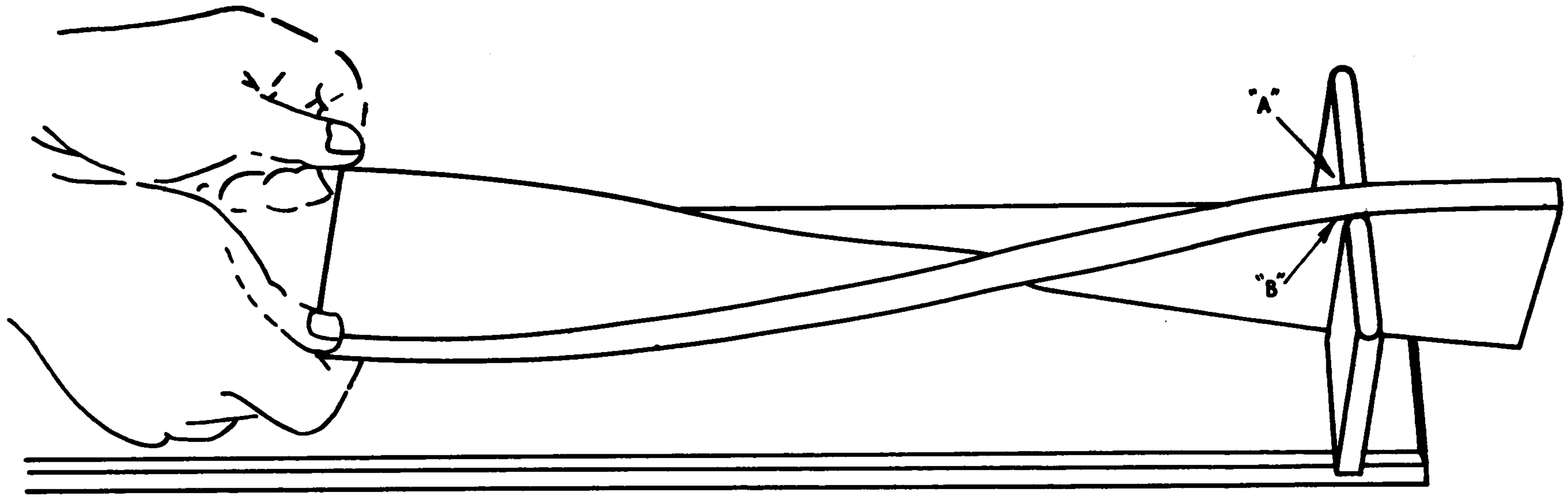



FIG. 7. ACTION OF SPRING END IN A CONVENTIONAL ANCHOR.

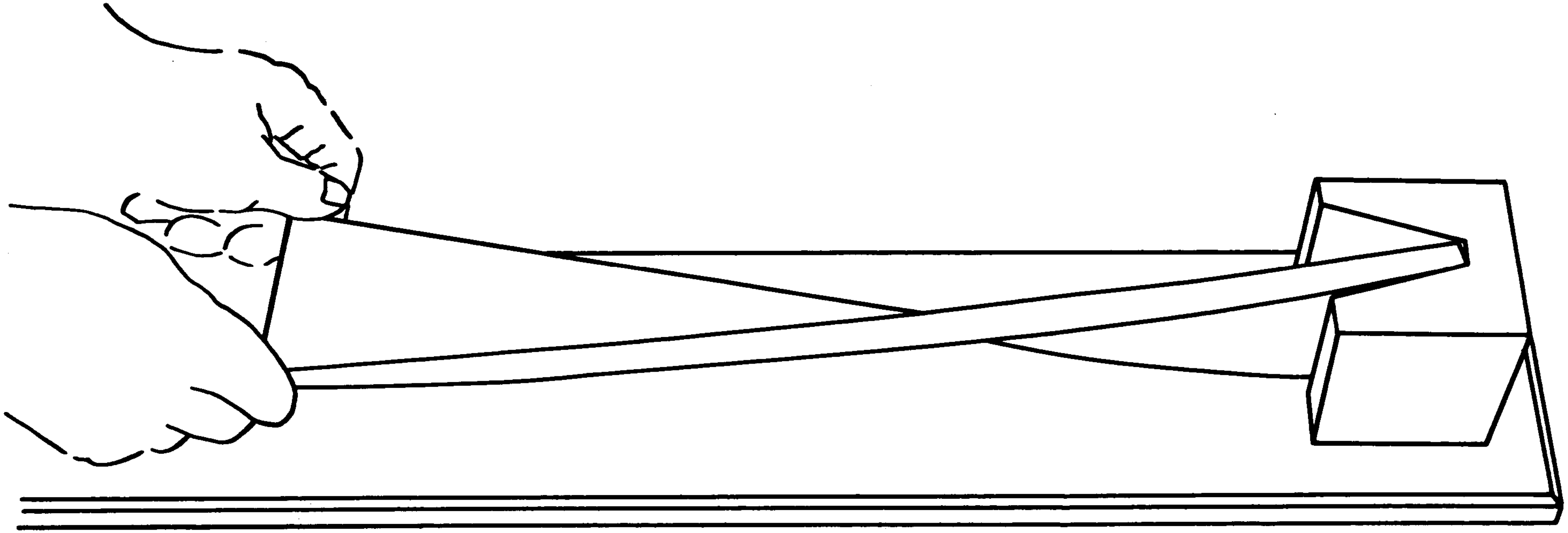


FIG. 8. ACTION OF SPRING END IN V-NOTCH.

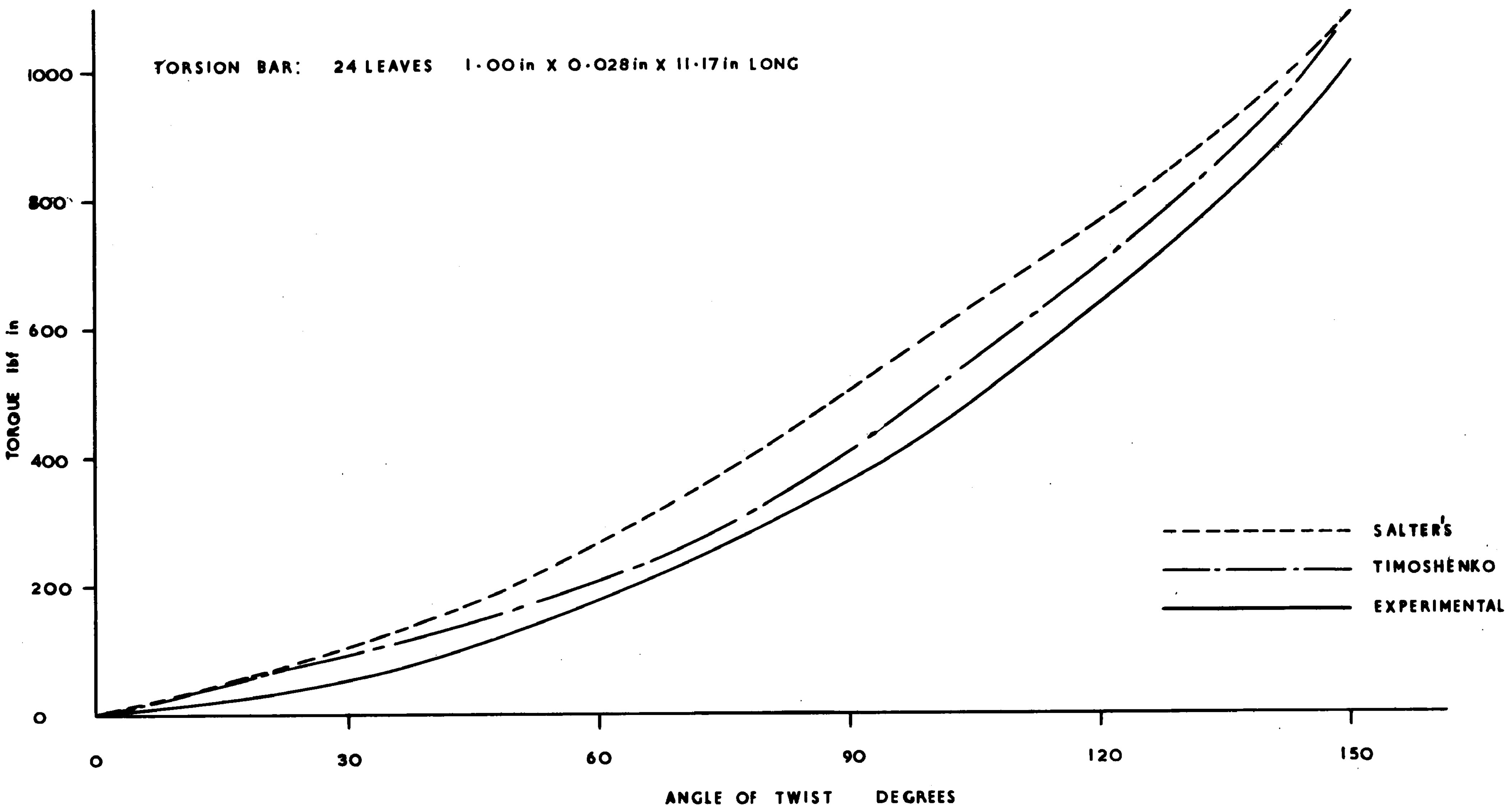

TORSION BAR: 24 LEAVES 1·00 in X 0·028 in X 11·17 in LONG
1000
800
600
400
200
0
TORQUE lbf in
0
30
60
90
120
150
ANGLE OF TWIST DEGREES
SALTER'S
TIMOSHENKO
EXPERIMENTAL

FIG. 9.

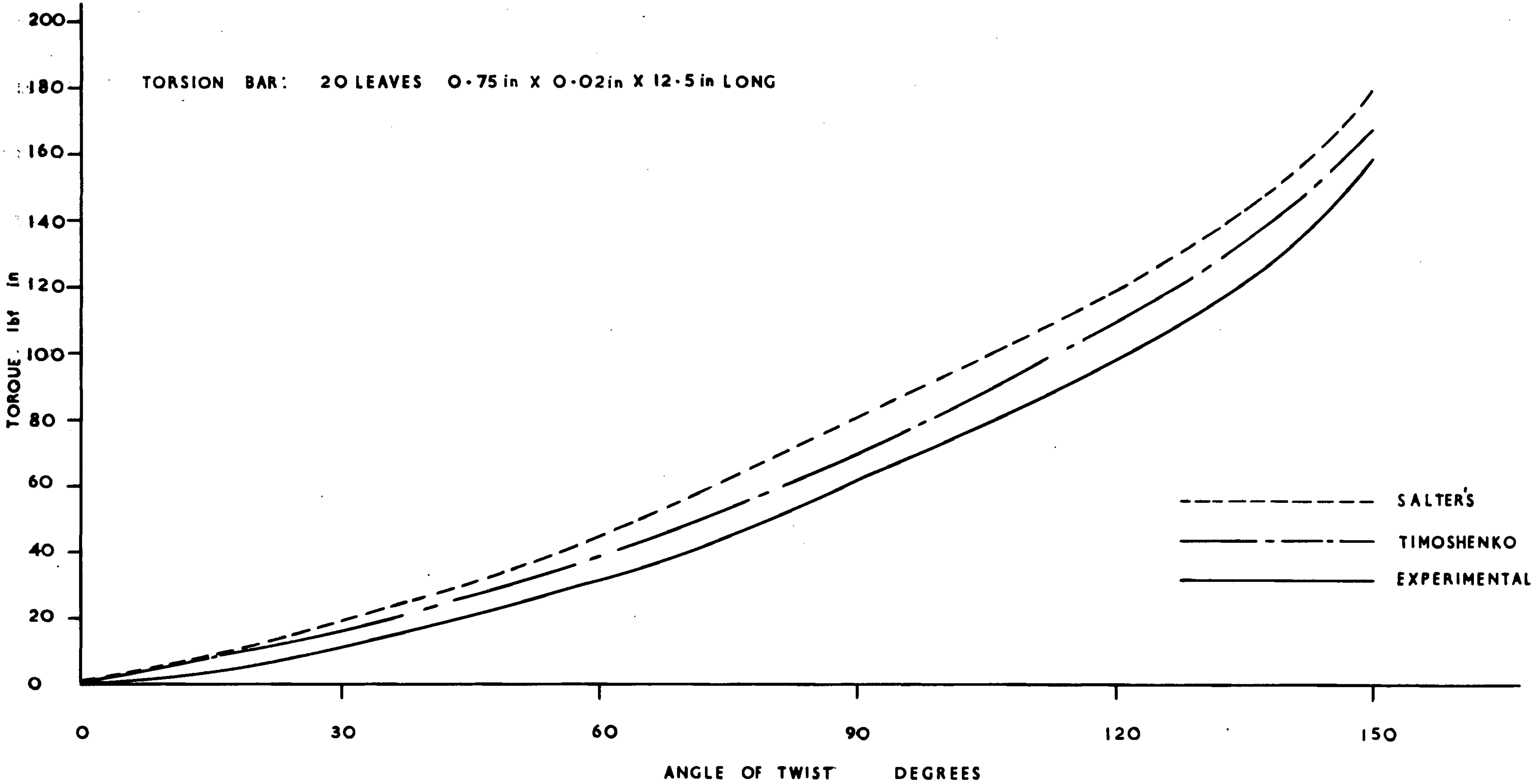

TORSION BAR: 20 LEAVES 0·75 in X 0·02 in X 12·5 in LONG
TORQUE. lbf in
200
180
160
140
120
100
80
60
40
20
0
30
60
90
120
150
SALTER'S
TIMOSHENKO
EXPERIMENTAL
ANGLE OF TWIST DEGREES

FIG. 10.

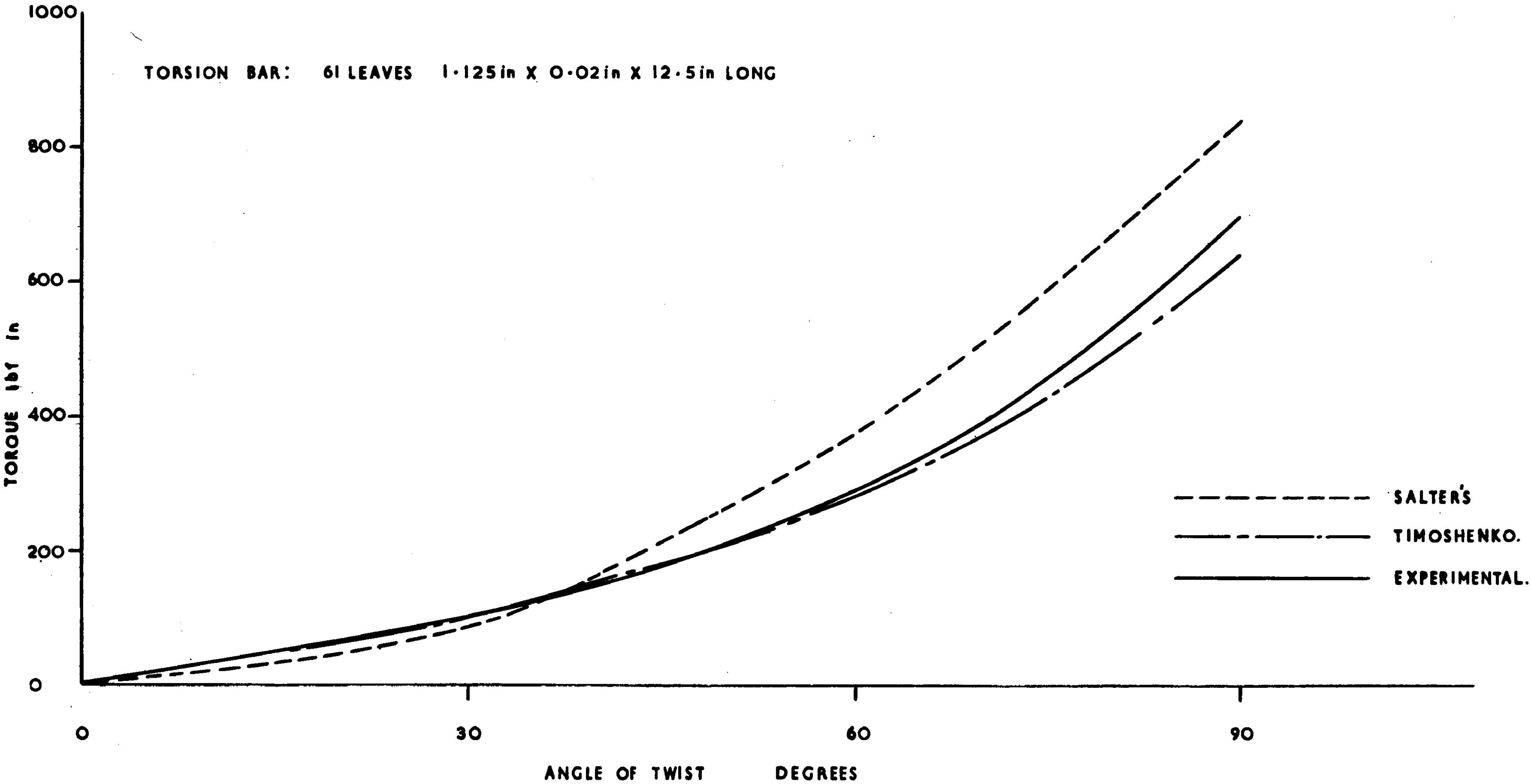

TORSION BAR: 61 LEAVES 1·125in X 0·02in X 12·5in LONG
1000
800
600
400
200
0
TORQUE lbf in
0
30
60
90
ANGLE OF TWIST DEGREES
SALTER'S
TIMOSHENKO.
EXPERIMENTAL.

FIG. 11.

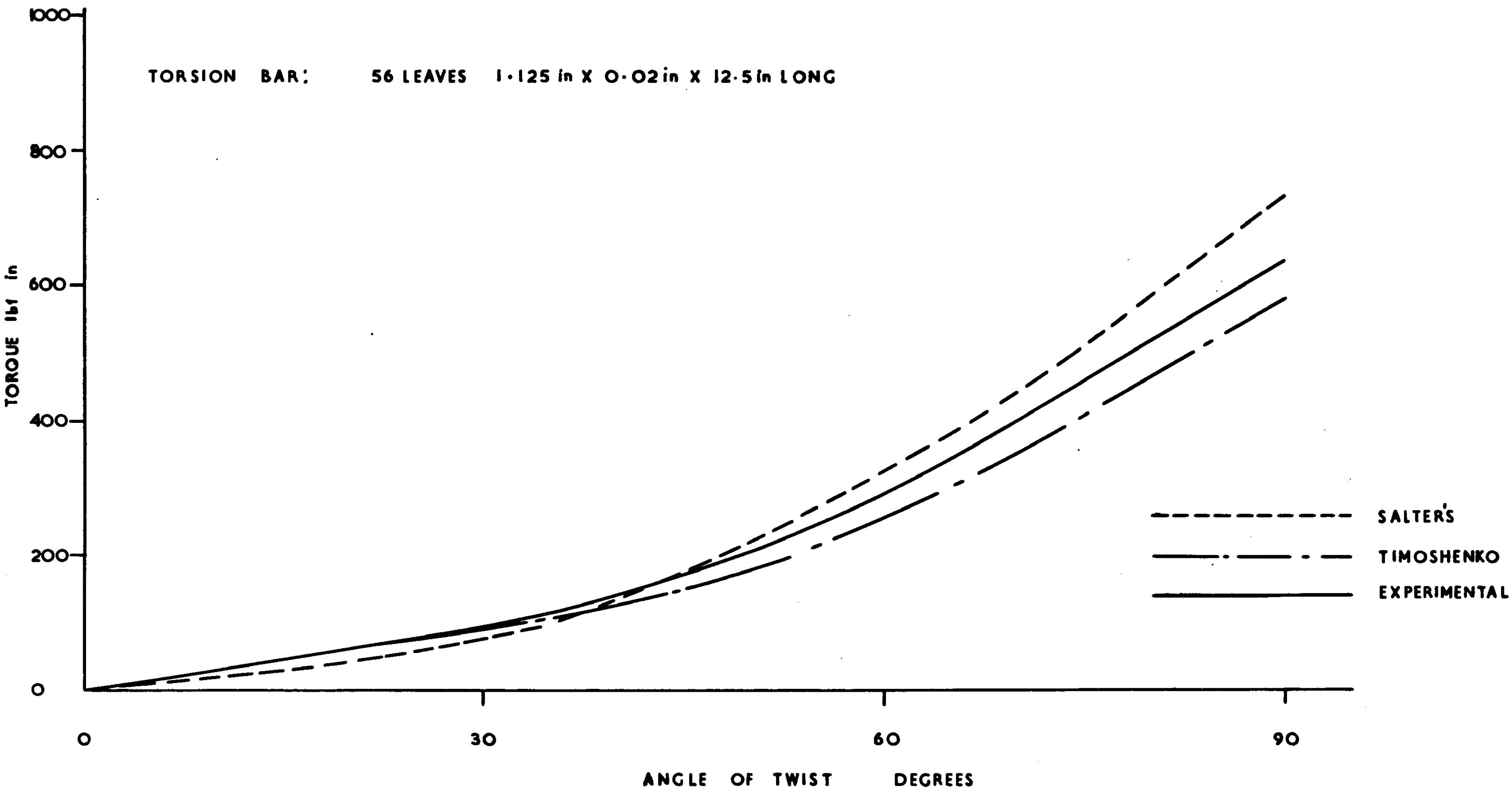
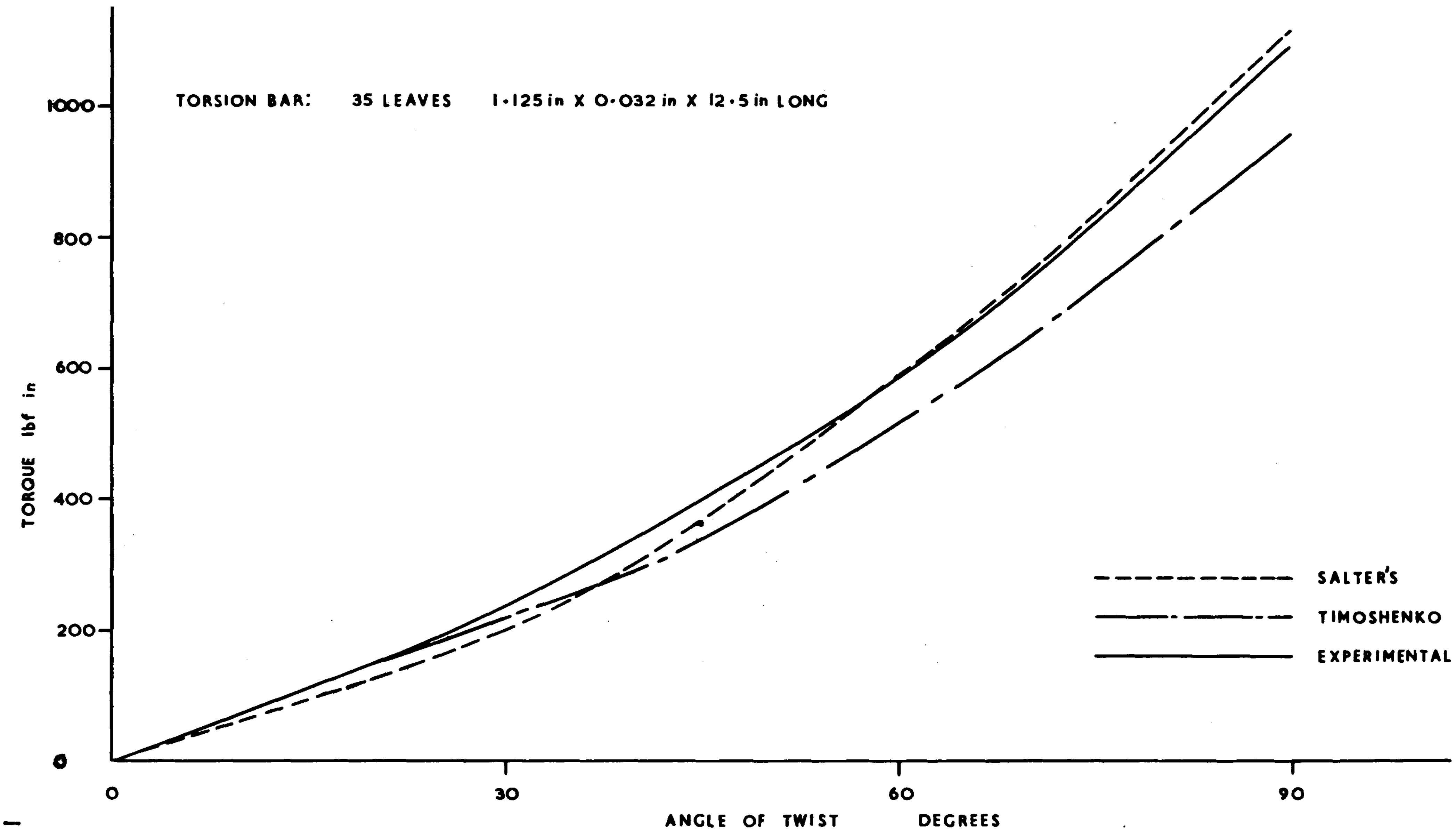

TORSION BAR: 56 LEAVES 1·125 in X 0·02 in X 12·5 in LONG
1000
800
600
400
200
0
TORQUE lbf in
0
30
60
90
ANGLE OF TWIST DEGREES
SALTER'S
TIMOSHENKO
EXPERIMENTAL

FIG. 12.



TORSION BAR: 35 LEAVES 1·125 in X 0·032 in X 12·5 in LONG
TORQUE lbf in
1000
800
600
400
200
0
0
30
60
90
ANGLE OF TWIST DEGREES
SALTER'S
TIMOSHENKO
EXPERIMENTAL

FIG. 13.

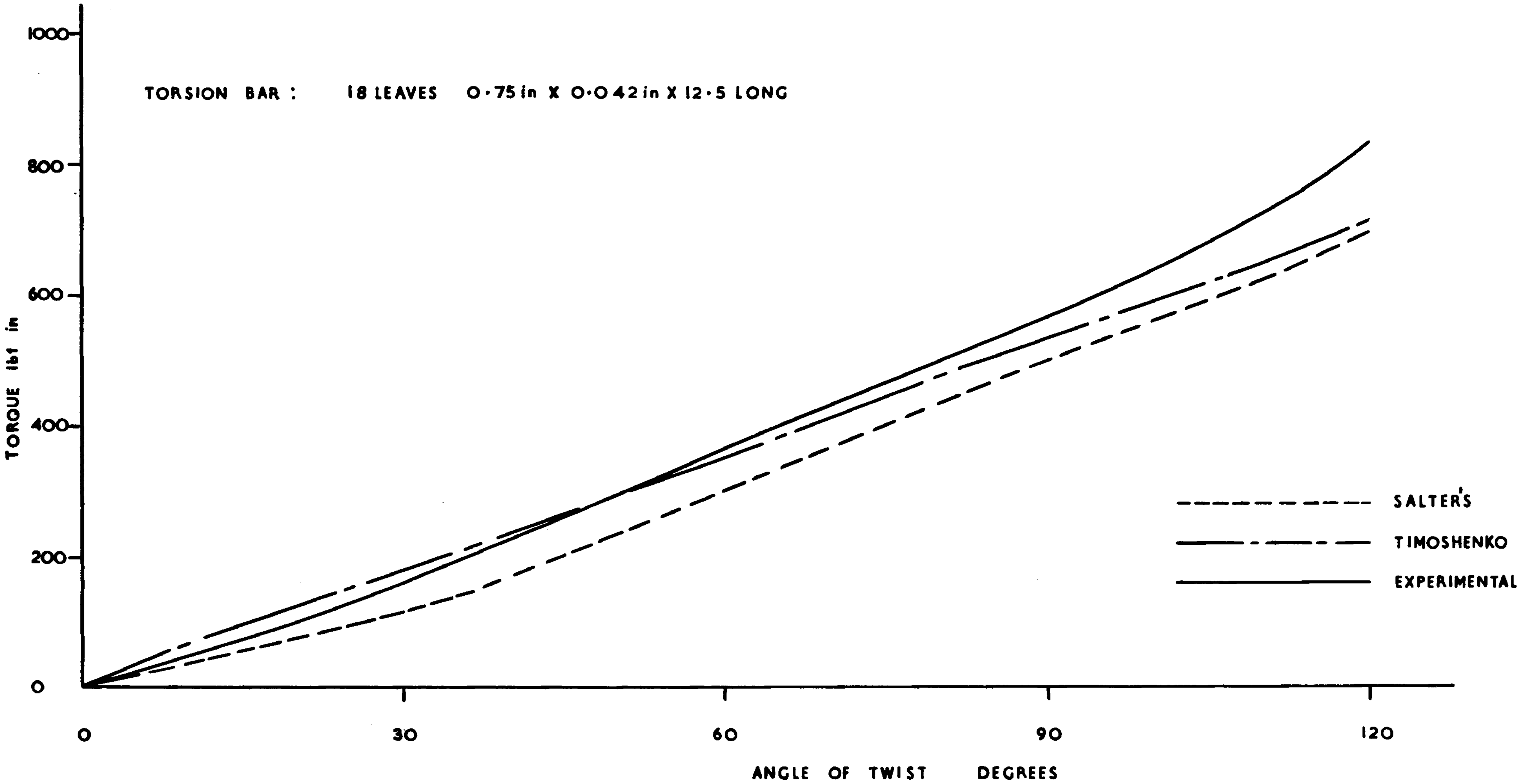

TORSION BAR : 18 LEAVES 0·75 in X 0·042 in X 12·5 LONG
1000
800
600
400
200
0
TORQUE lbf in
0
30
60
90
120
ANGLE OF TWIST DEGREES
SALTER'S
TIMOSHENKO
EXPERIMENTAL

FIG. 14.

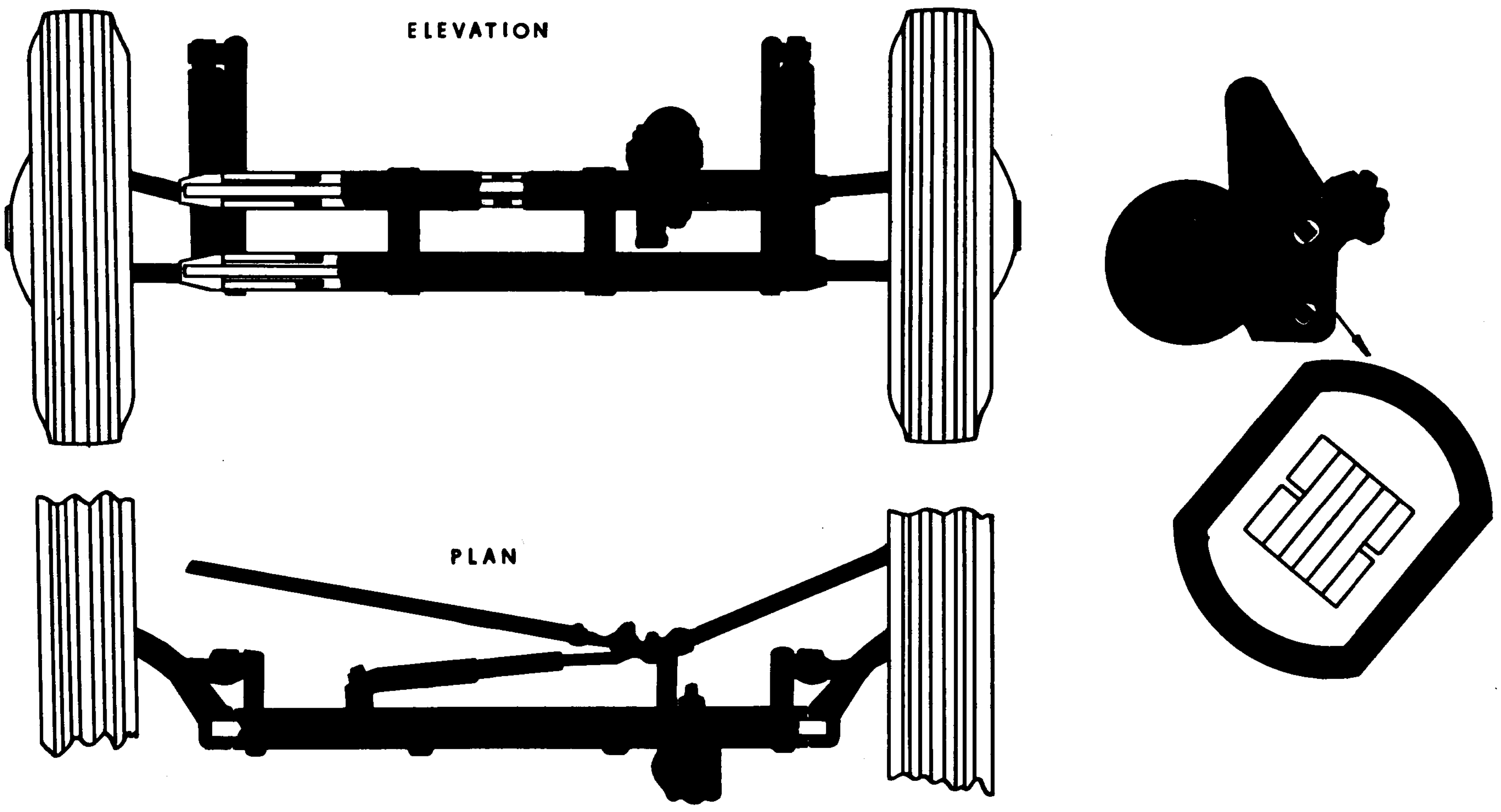



FIG. 15. VOLKSWAGEN FRONT SUSPENSION.