THE SPRING RESEARCH AND MANUFACTURERS' ASSOCIATION

STATISTICAL ANALYSIS OF FATIGUE DATA PRODUCED FROM COMPRESSION SPRINGS

by

J. Desforges, B.Met.

Report No. 274

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SUMMARY

The purpose of this investigation is to explore statistical methods for the analysis of fatigue data for springs. The material and condition of the material used in this investigation was selected in order to produce data for the analysis rather than to produce design data for use by the spring industry.

Limited life and infinite life fatigue data for BS 5216 HD3 compression springs have been determined and the results analysed by a number of statistical methods.

For any selected test stress level, it has been shown that the finite life data are not distributed in a log-normal manner. However, by taking sufficient samples at any particular stress and ignoring the results beyond the median, analysis by the 'Weibull' technique and also by 'normal' statistical methods was possible and gave similar 5% and 50% probabilities of failure.

Probit analysis of the data close to the fatigue limit has enabled probabilities of failure to be determined, thus enabling a more precise measure of the fatigue limit of springs to be made.

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1. INTRODUCTION

Results obtained by fatigue testing a group of springs have very little value until they have been analysed statistically. Correct statistical analysis of the data obtained allows predictions to be made of the expected life of other, similar springs.

In order to carry out a statistical analysis of the data, it is first necessary to identify the shape of the distribution of results. For simplicity it is generally assumed that the distribution of limited life data in fatigue is log-normal about the mean life; the analysis of all fatigue data produced at SRAMA is based on this assumption. It has been suggested by Weibull (1) that another pattern of distribution fits some fatigue data more accurately and this has become known as the Weibull distribution. In this report these two methods of analysis are compared in order to ascertain which is the more appropriate for use in analysis of limited life fatigue data for springs.

In many previous investigations of fatigue properties, the fatigue limit has been fixed by a quite arbitrary method involving only about three or four springs which survived 10^7 cycles. Although economical in terms of testing time and springs used, no statistical confidence can be placed in such data. However, by testing appreciably more specimens around this 'fatigue limit' it is possible to determine the probability of failure ~~or~~ survival and thereby obtain a more precise measure of the fatigue limit.

Various methods of testing and analysis are available for obtaining these probabilities of failure and one of these, the Probit method, is described in this report.

2. MATERIALS

2.1 Composition

The springs used for this work were made from wire to BS 5216:1975, grade HD3. The actual composition of this wire, together with the specified composition are given in Table I.

2.2 Spring Design

The spring design is given in Table II.

2.3 Mechanical Properties

The mechanical properties of the wire in the 'as received' condition and after a low temperature heat treatment at 350°C are given in Table III.

3. FATIGUE TESTING

Springs were tested on a valve spring fatigue testing machine in the unpeened condition, at initial stresses of 100 and 300 N/mm^2. About ten springs were tested initially for each initial stress, to give approximate S/N curves. From these, appropriate maximum stress values were chosen and a number of springs were tested over each stress range to allow analysis of the results by the Weibull and Probit methods.

4. ANALYSIS OF RESULTS

4.1 Least Squares Method

This method has been used extensively in the past by the Association for the purpose of determining the best fit straight line to describe all the limited life data when plotted on a logarithmic endurance scale and linear stress scale (i.e. S/N curve). The method involves calculating the smallest value for the sum of the squares of the distances between each experimental point and the line in terms of cycles. Such a line indicates 50% probability of failure. By assuming a log-normal distribution of endurance, other probability levels of failure can be placed on the data and a 5% probability is often shown in SRAMA reports.

In practice, however, not all fatigue data fits a log-normal distribution. At the higher levels of test stress a log-normal distribution may be appropriate but as the test stress decreases to approach the fatigue limit, the distribution of endurance becomes increasingly more skew with consequently more error in the 5% probability of failure prediction. Fortunately, the estimate of a 5% probability of failure based on a normal distribution when, in fact, a skew distribution exists results in a conservative figure.

4.2 Weibull Method of Analysis

In order to analyse fatigue data by the Weibull method of analysis it is necessary to test a number of springs over each stress range. The number of tests carried out at each stress range determines the amount of confidence which can be put on the results. For this work a sample size of twenty was chosen, although, it is claimed, as few as eight specimens may be used.

Each stress range gives a point on the S/N curve. A minimum of two maximum stresses for each initial stress are therefore needed to produce the limited life portion of the S/N curve. For this work three maximum stresses were chosen for each initial stress to enable the S/N curve to be constructed more accurately. About ten springs were tested initially to give an approximate S/N curve. From this the three maximum stresses were chosen, equally spaced on the limited life portion of the curve, and twenty springs were fatigue tested at each. The maximum stresses were 840, 920 and 1000 N/mm^2 for an initial stress of 100 N/mm^2, and 980, 1040 and 1100 for an initial stress of 300 N/mm^2.

When the springs had been fatigue tested, the results obtained for each stress range were ranked in order of increasing life. Each life was then assigned a 'Median Rank', obtained from tables in the literature(2), which represents the cumulative percentage failure. Tables IV and V give the results obtained over each stress range along with the appropriate median ranks.

These results were then plotted on Weibull Probability Paper as shown in Figs. 1 and 2. This paper has logarithm of life as the abscissa and $\log_e \log_e \left(\frac{100}{100-F}\right)$ as the ordinate, where F is the median rank of the item under consideration. If the results do fit a Weibull distribution, the graph obtained by plotting median rank against life should approximate to a straight line. It was found for the data obtained in these tests that only the results for the lower lives fell on a straight line. A few springs in most stress ranges survived many more cycles than the others and these had the effect of changing the straight line to a curve at longer lives. As it is the possibility of early failure that is of most interest to the spring industry, it was considered valid to use only the ten lowest results and the best fit straight line was drawn through these ten points for each stress range.

From the Weibull plot, the life at which the springs have any given probability of failure can be found. For example, from the line for the stress range 100 to 1000 N/mm^2 in Fig. 1, it was found that 5% of springs are likely to fail at less than 72,000 cycles and 50% at less than 148,000 cycles. Similarly, the lives for 5% and 50% probability of failure known here as the S_5 and S_{50} lives, were found for the other stress ranges. The results obtained were plotted on S/N curves (Figs. 3 and 4) and the limited life portion of the S/N curve was constructed for the S_5 and S_{50} lives.

The question arises of how confident can one be that these S/N curves are truly representative of the springs from which the test specimens were taken. The more specimens tested over each stress range, the more accurate the estimate of the S_5 and S_{50} life and therefore the greater the confidence that can be placed in the S/N curve produced from the data. Using Weibull analysis, confidence intervals can be placed on any given life.

The 90% confidence interval for the S_5 life, for example, means in practical terms that if many groups of springs were tested, resulting in various estimates of the S_5 life, 90 out of every 100 of the S_5 life predictions would be likely to fall within this interval. Since the concern is with the possibilities of failure before the S_5 life prediction it is only necessary to find the lower limit of the 90% confidence interval. This gives an estimate of the minimum life for 95% probability of survival at 95% confidence.

The lower limit of the 90% confidence interval is obtained from the S_5 by multiplying by a reduction factor. The value of this reduction factor depends upon the slope of the Weibull plot, b, (a measure of the scatter in the results) and the sample size, N. In order to calculate the Weibull slope it must be remembered that the vertical axis of Weibull Probability Paper represents $\log_e \log_e (\frac{100}{100-F})$ and the horizontal axis represents $\log_e$ (life). The slope of the Weibull plot for the stress range 100 to 840 N/mm^2 is $\frac{AB}{BC}$ in Fig. 1.

$$\text{i.e. } b = \frac{\log_e \log_e (\frac{100}{100 - 40}) - \log_e \log_e (\frac{100}{100 - 5})}{\log_e (260{,}000) - \log_e (170{,}000)}$$

$$= 5.39$$

Fig. 5 gives the reduction factor for the lower limit of the 90% confidence interval for various Weibull slopes and a sample size of 20. This curve was derived according to a method described in the literature(2). For the slope of 5.39 obtained for the stress range 100 to 840 N/mm^2, the reduction factor is found to be 0.735. The S_5 life of 170,000 was multiplied by this reduction factor giving the lower limit of the 90% confidence interval at 125,000. This procedure was repeated for the other stress ranges and the lower limit of the 90% confidence interval for the S_5 life was then constructed on the S/N curve.

4.3 Graphical Normal Distribution Analysis

The results of the fatigue tests given in Table IV and V can also be analysed graphically assuming a log-normal distribution. The logarithm of the number of cycles to failure, ranked in order of increasing magnitude are plotted against cumulative percentage failure on normal probability paper (Figs. 6 and 7). If the points are found to approximate to a straight line the results can be considered to follow a log-normal distribution.

As was the case for the Weibull plots, it was found that only the experimental results with the lower lives fell on an approximate straight line and therefore only these were used in the analysis. The life for 5% and 50% probability of failure were found from the graphs and plotted on S/N curves (Figs. 8 and 9). With this method of analysis, confidence levels cannot be placed on the values of 5% and 50% probability of failure.

With reference to the limited life fatigue data as plotted as a Weibull distribution in Figs. 1 and 2 and as a normal distribution in Figs. 6 and 7, a difference in the form of the graphs obtained can be observed between the data obtained with an initial stress of 100 N/mm^2 in Figs. 1 and 6 and at the higher initial stress in Figs. 2 and 7. At the lower initial stress, the lines do not cross and hence as may be expected the ranking order for failure is the same at the 5% and 50% probabilities of failure.

In the case of springs tested with an initial stress of 300 N/mm^2, it can be seen in Figs. 2 and 7 that the lines (which are a 'best-fit' through the data for the first half of the specimens) for maximum stresses of 1100 N/mm^2 and 1040 N/mm^2 intersect. The reason for this is that the life of the first broken specimen when tested to 1040 N/mm^2 was less than the first at either of the other two stress levels. (This can be most clearly seen in Tavle V.) Since we have no reason for excluding this result, it would seem that for a 5% failure level the life with a maximum stress of 1100 N/mm^2 is greater than that for springs with a maximum stress of 1040 N/mm^2, a point which can be seen in the S/N curves in Figs. 4 and 9. It will be interesting to see if this is a freak result or one that recurs when other data have been obtained and treated by the same analysis.

4.4 Probit Method of Analysis

When infinite life data are plotted on an S/N curve, the fatigue limit is usually constructed at a stress just below the last failure before a given number of cycles, usually 10^7. This may give the false impression that all springs tested above this limit will fail and all those tested at lower stresses will have infinite life. To avoid confusion it would be more useful if the stress for any given probability of failure could be plotted. The Probit method of analysis(3) enables this to be done and is used to determine the stresses for 5% and 50% probability of failure.

A few springs were initially tested around the estimated fatigue limit in order to obtain an idea of the stress at which 50% specimens would be likely to fail before reaching 10^7 cycles endurance.

To satisfy the Probit method of analysis(3) it is necessary to select a number of stress levels of equal increment or decrement around the expected 50% failure stress. Based on the pattern of results emerging from the initial tests conducted around the fatigue limit, a 40 N/mm^2 stress interval was chosen for these subsequent tests.

Two stress levels were chosen, equally spaced, above the expected 50% failure stress and to facilitate the calculation of the low level of probability of failure (i.e. 0.05), three levels of stress were employed below the estimated 50% failure stress. Six specimens were fatigue tested up to 10^7 cycles, at the approximate 50% level and the number of specimens tested at the other stresses was determined according to Table VI(3). If the groups for each stress were of equal size, then the results would have to be weighted for analysis to account for differences in precision of results at each level. By weighting the groups of test specimens, the analysis is simplified. A minimum of 50 springs must be tested to obtain a reasonable degree of accuracy.

The results of the fatigue tests for initial stresses of 100 and 300 N/mm^2 are given in graphical form in Figs. 10 and 11. The percentage of specimens which survived both 10^6 and 10^7 cycles were calculated as shown in Tables VII and VIII, and plotted against stress on normal probability paper (Figs. 12 and 13). The stresses for 5% and 50% probability of failure at 10^6 and 10^7 cycles were found from this graph and these were plotted along with the data produced by Weibull analysis on Figs. 3 and 4, and with the normal distribution analysis in Figs. 8 and 9.

It is the normal practice to represent S/N curves for ferrous materials by two straight lines. In this instance, however, a curve has been drawn through the S_5 and S_{50} points. It is not recommended at this stage that the curve should be used for design purposes but rather to indicate that a smooth curve is a more likely representation for the early failure fatigue behaviour compared with the "50% failure" behaviour which is traditionally represented as a straight line over the limited life range with a horizontal fatigue limit.

Modified Goodman diagrams (Figs. 14 and 15) for 10^6 and 10^7 cycles have also been produced from the Probit data.

5. DISCUSSION

It was found in the analysis of the results that neither the Weibull nor the log-normal distribution describes all the data obtained. The lower life results do, however, fit both distributions fairly accurately. The S/N curves obtained by assuming either a Weibull or a log-normal distribution were very similar, which suggests that the Weibull distribution approximates to a log-normal distribution up to the mean. Either distribution therefore seems to be equally suitable for these lower lives, but as confidence intervals can only be obtained by using the Weibull distribution, this method obviously had an advantage.

The main disadvantages of limited life data analysis by either of the methods described in this report is that a large number of test springs are required. If the method of least squares is used, only about 20 springs are needed to produce an S/N curve but more than 60 springs were required to produce just the limited life portion of the S/N curve using the Weibull technique. This is also the disadvantage of the Probit method of analysis which gives a lot of information about the infinite life portion of the S/N curve. This again involves more fatigue testing than used in producing the fatigue limit in the usual way.

To produce a complete S/N curve by the Weibull and Probit techniques, more than 100 springs are required and this results in an increase in testing time and in the effort required for analysing the results. This is justified, however, by the increase in the amount of information obtained and the accuracy of the information. The S/N curves and Goodman diagrams produced in this way give the spring manufacturer and user a far better guide for choosing allowable working stresses in the design of components.

6. CONCLUSIONS

1. Neither a Weibull nor a log-normal distribution was followed by all the fatigue data collected for this report. However, the results for lives below the median did follow both distributions.

2. Analysis of the results assuming either distribution for the lower lives gave similar S/N curves.

3. Using the Weibull method of analysis, confidence intervals may be placed on any required probability of failure.

4. Analysis by the method of least squares will give a less accurate S/N curve.

5. Analysis of infinite life data, carried out by the Probit method, enabled the stresses for 5% and 50% probability of failure to be found.

7. REFERENCES

1. WEIBULL W. "Fatigue Testing and Analysis of Results" Pergamon Press, 1961.

2. JOHNSON L.C. "The Statistical Treatment of Fatigue Experiments". Elsevier Publishing Co., 1964.

3. "Methods of Fatigue Testing". British Standard BS 3518:Part 5:1966.

TABLE I COMPOSITION

	C%	Si%	Mn%	S%	P%
Actual	0.76	0.30	0.57	0.016	0.010
Specified	0.55 -0.85	0.35 max	0.30 -1.00	0.030 max	0.030 max

TABLE II SPRING DESIGN

Wire Diameter (mm)	2.65
Mean Coil Diameter (mm)	21.7
No. of Active Coils	3.5
Total No. of Coils	5.5
Free Length After End Grinding, L.T.H.T. and Prestressing (mm)	41.1
Solid Stress after Prestressing N/mm^2	1254

TABLE III MECHANICAL PROPERTIES

	Tensile Strength N/mm^2	0.05% Proof Stress N/mm^2	0.1% Proof Stress N/mm^2	Limit of Proportionality N/mm^2
As Received	1750	940	1150	580
L.T.H.T. 350°C	1825	1485	1550	1240

TABLE IV LIMITED LIFE FATIGUE DATA FOR AN INITIAL STRESS OF 100 N/mm^2

Order Number	Cycles to Failure			Median Rank
	Max. Stress 840 N/mm^2	Max. Stress 920 N/mm^2	Max. Stress 1000 N/mm^2	
1	171,000	100,000	63,000	3.41
2	181,000	107,000	86,000	8.31
3	202,000	119,000	90,000	13.22
4	202,000	144,000	108,000	18.12
5	224,000	183,000	113,000	23.02
6	237,000	183,000	125,000	27.03
7	243,000	211,000	126,000	32.82
8	252,000	215,000	133,000	37.74
9	280,000	216,000	134,000	42.64
10	297,000	237,000	145,000	47.55
11	300,000	237,000	149,000	52.45
12	315,000	238,000	154,000	57.36
13	325,000	270,000	155,000	62.26
14	372,000	272,000	156,000	67.17
15	477,000	274,000	157,000	72.07
16	1,428,000	292,000	157,000	76.98
17	1,941,000	309,000	161,000	81.88
18	1,966,000	333,000	180,000	86.78
19	3,037,000	378,000	234,000	91.69
20	3,348,000	2,970,000	262,000	96.59

TABLE V LIMITED LIFE FATIGUE DATA FOR AN INITIAL STRESS OF 300 N/mm^2

Order Number	CYCLES TO FAILURE			Median Rank
	Max. Stress 980 N/mm2	Max. Stress 1040 N/mm^2	Max. Stress 1100 N/mm^2	
1	175,000	72,000	136,000	3.41
2	183,000	175,000	140,000	8.31
3	240,000	199,000	145,000	13.22
4	263,000	202,000	163,000	18.12
5	369,000	221,000	180,000	23.02
6	440,000	247,000	184,000	27.93
7	486,000	254,000	197,000	32.82
8	490,000	256,000	202,000	37.74
9	639,000	268,000	230,000	42.64
10	890,000	317,000	238,000	47.55
11	1,450,000	333,000	240,000	52.45
12	1,804,000	368,000	248,000	57.36
13	2,000,000 U/B	373,000	250,000	62.26
14	2,000,000 U/B	406,000	279,000	67.17
15	2,000,000 U/B	423,000	281,000	72.07
16	2,000,000 U/B	504,000	288,000	76.98
17	2,000,000 U/B	549,000	294,000	81.88
18	10,000,000 U/B	594,000	297,000	86.78
19	10,000,000 U/B	3,038,000	333,000	91.69
20	10,000,000 U/B	3,580,000	369,000	96.59

TABLE VI ALLOCATION OF TEST SPECIMENS FOR PROBIT METHOD OF ANALYSIS

Expected Percentage Failures	Relative Group Size
25 to 75	1
15 to 20 or 80 to 85	1.5
10 or 90	2
5 or 95	3
2 or 98	5

TABLE VII RESULTS OF PROBIT TESTS FOR AN INITIAL STRESS OF 100 N/mm^2

Max. Stress N/mm^2	No. of Specimens	Number Unbroken at 10^6 Cycles	% Unbroken at 10^6 Cycles	Number Unbroken at 10^7 Cycles	% Unbroken at 10^7 Cycles
840	12	3	25.0	0	0
800	9	6	66.7	3	33.3
760	6	4	66.7	4	66.7
720	9	8	88.9	7	77.8
680	12	12	100	11	91.7
640	18	18	100	17	94.4

TABLE VIII RESULTS OF PROBIT TESTS FOR INITIAL STRESS OF 300 N/mm^2

Max. Stress N/mm^2	No. of Specimens	Number Unbroken at 10^6 Cycles	% Unbroken at 10^6 Cycles	Number Unbroken at 10^7 Cycles	% Unbroken at 10^7 Cycles
1020	12	3	25.0	0	0
980	9	4	44.4	3	33.3
940	6	3	50.0	3	50.0
900	9	8	88.9	7	77.8
860	12	11	91.7	10	83.3
820	18	17	94.4	17	94.4

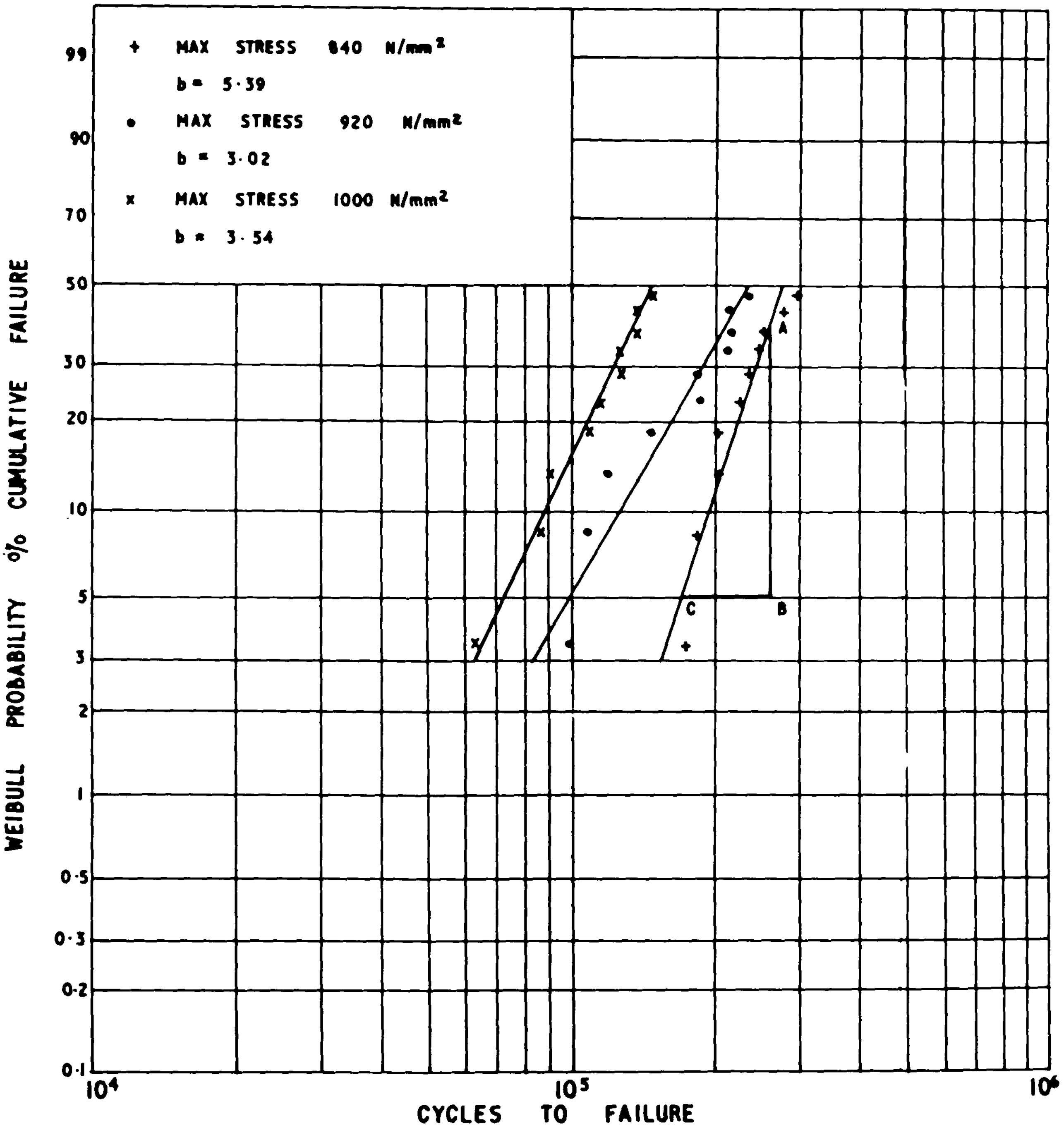

FIG. 1 WEIBULL PLOT FOR AN INITIAL STRESS OF 100 N/mm^2

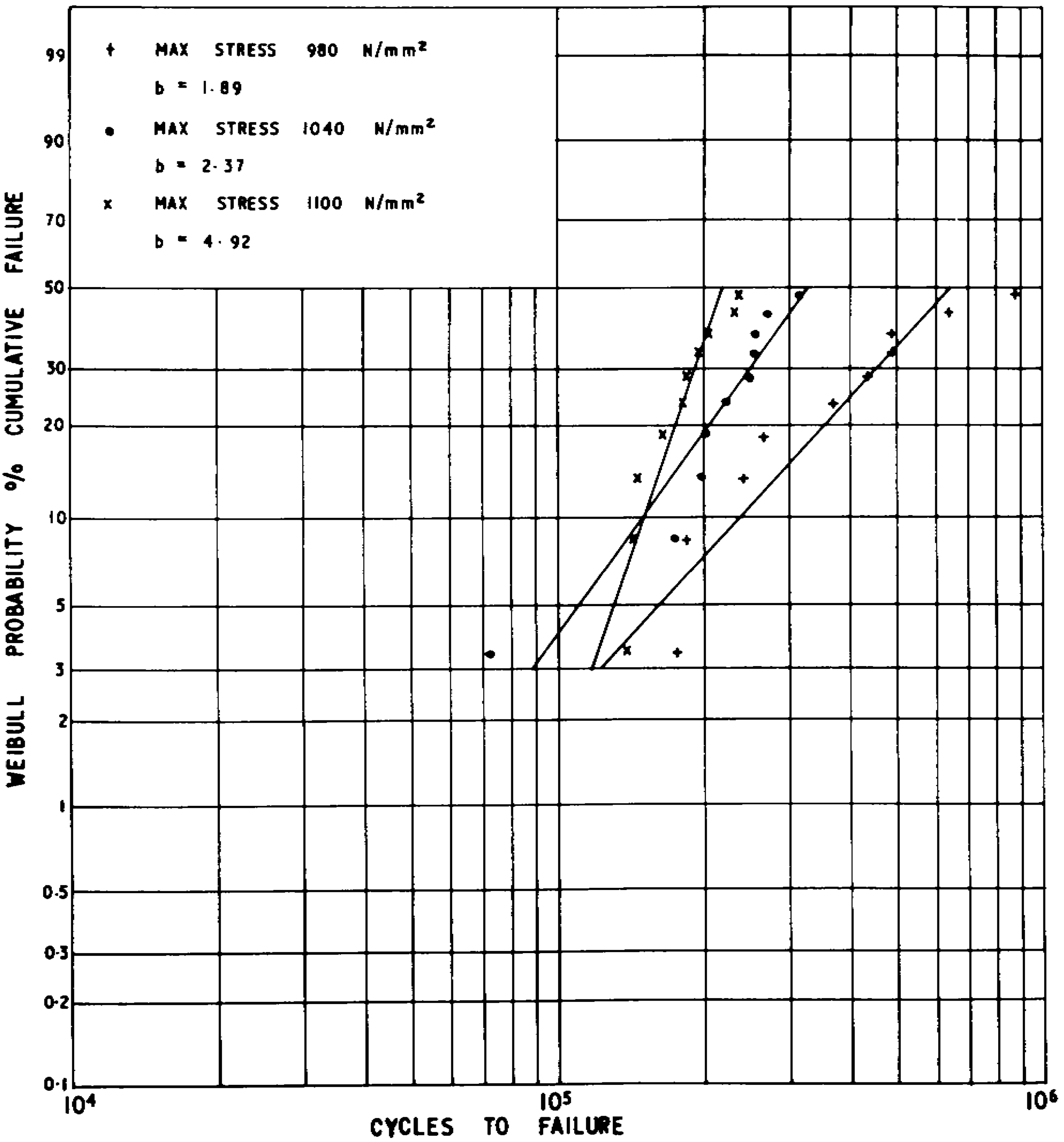

FIG. 2 WEIBULL PLOT FOR AN INITIAL STRESS OF 300 N/mm²

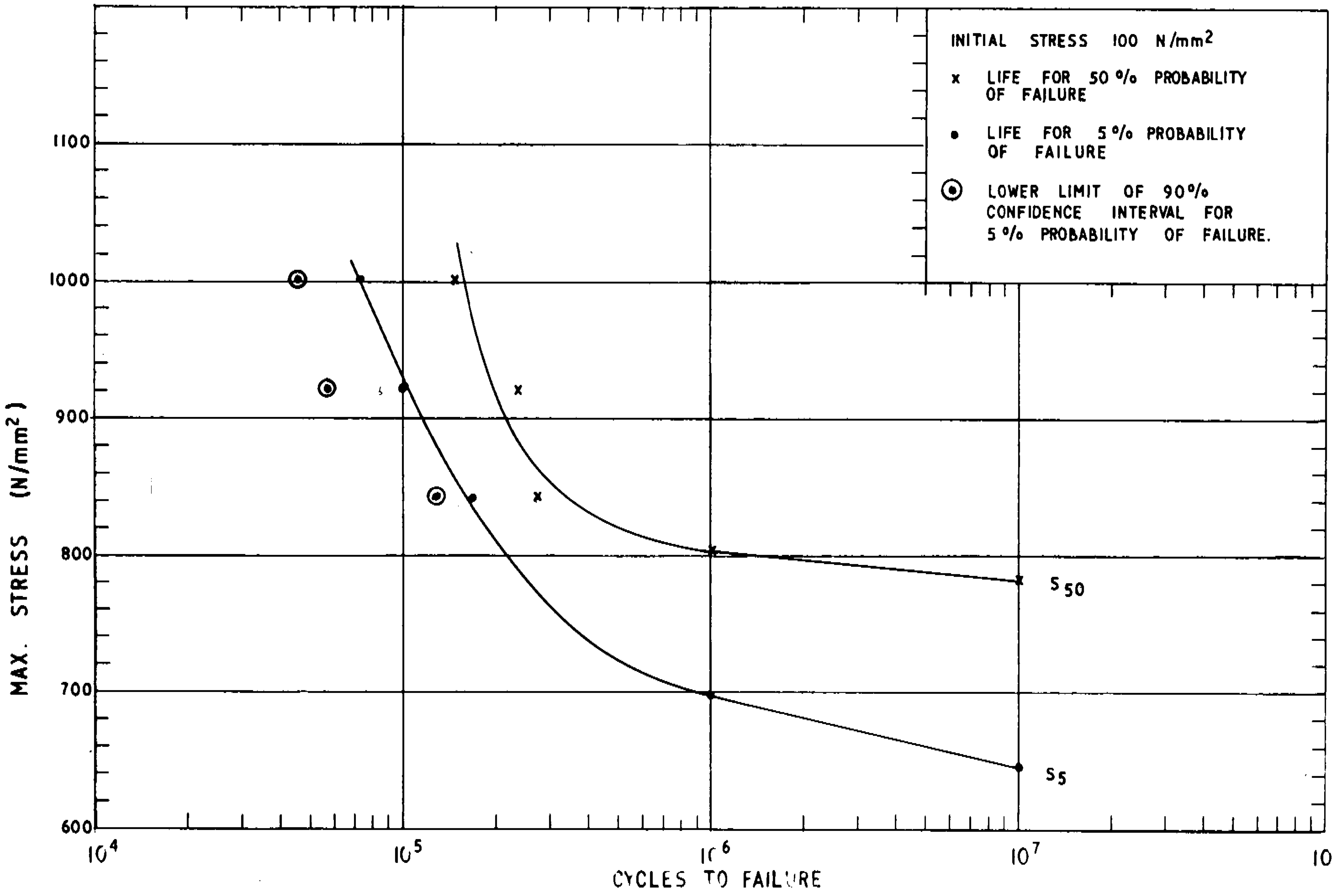

FIG. 3 S/N CURVE PRODUCED BY WEIBULL AND PROBIT ANALYSIS FOR AN INITIAL STRESS OF 100 N/mm²

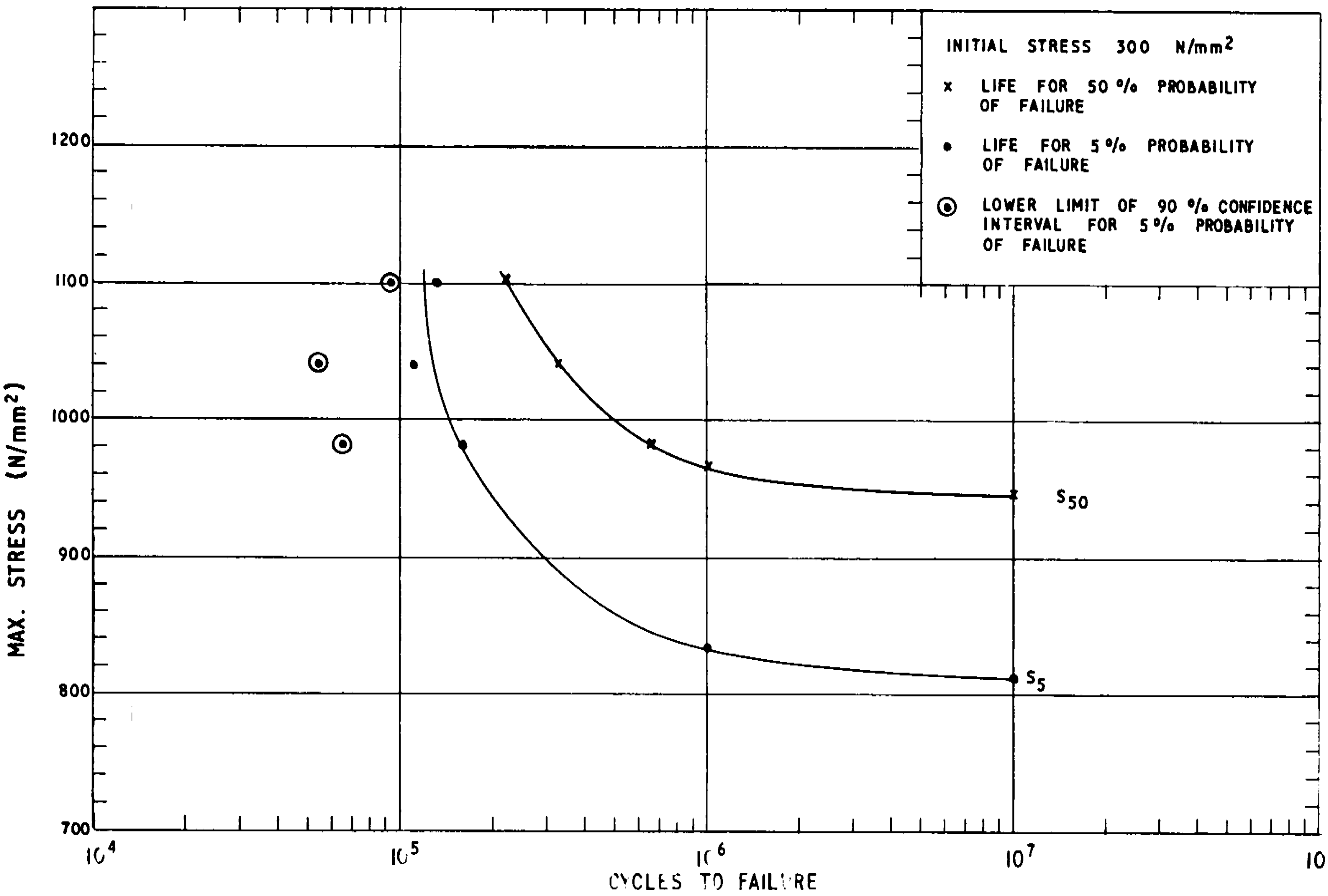

FIG. 4 S/N CURVE PRODUCED BY WEIBULL AND PROBIT ANALYSIS FOR AN INITIAL STRESS OF 300 N/mm²

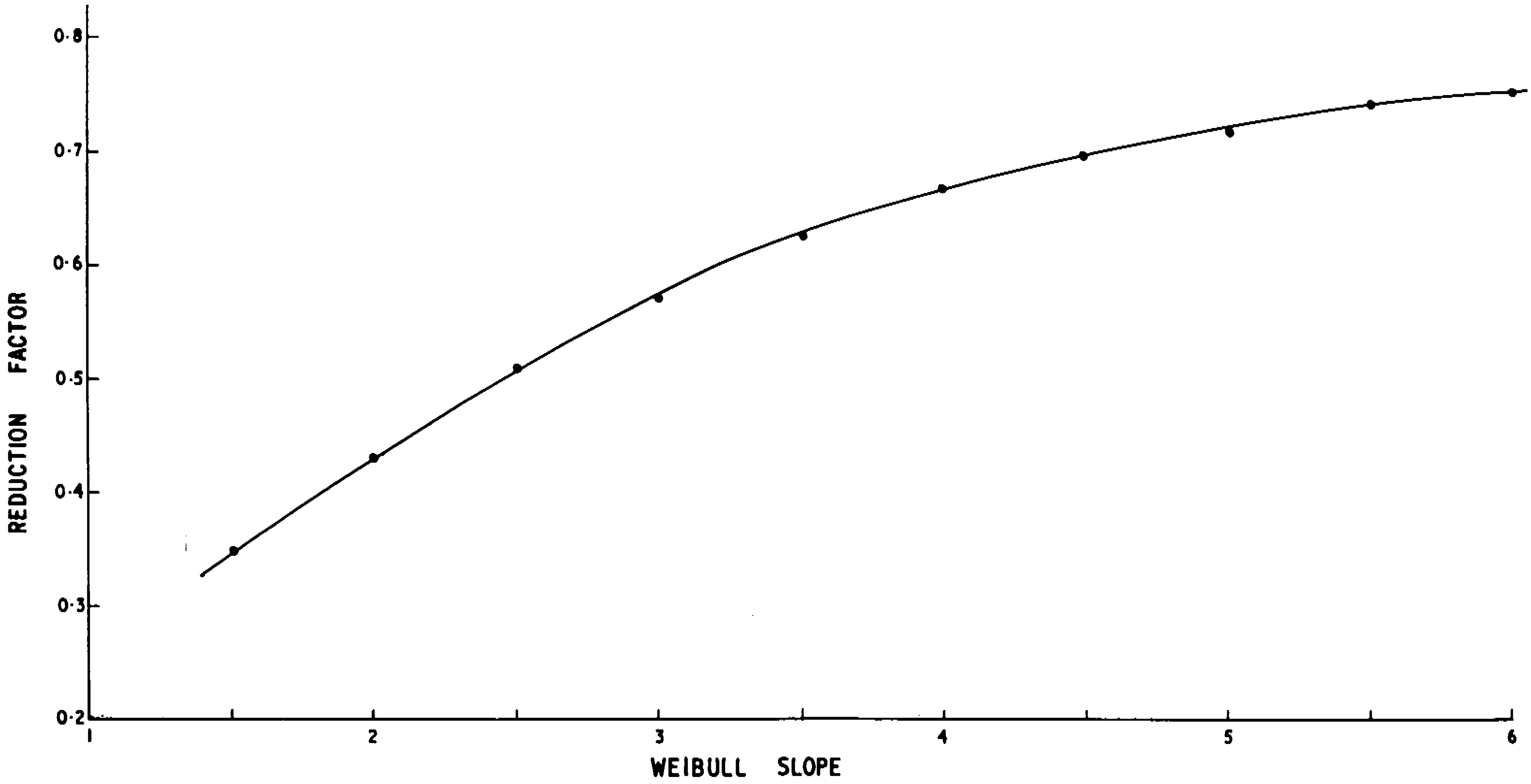

FIG. 5 REDUCTION FACTOR FOR CALCULATION OF THE LOWER LIMIT OF THE 90 % CONFIDENCE INTERVAL FOR S_5 LIFE FOR A SAMPLE SIZE OF 20

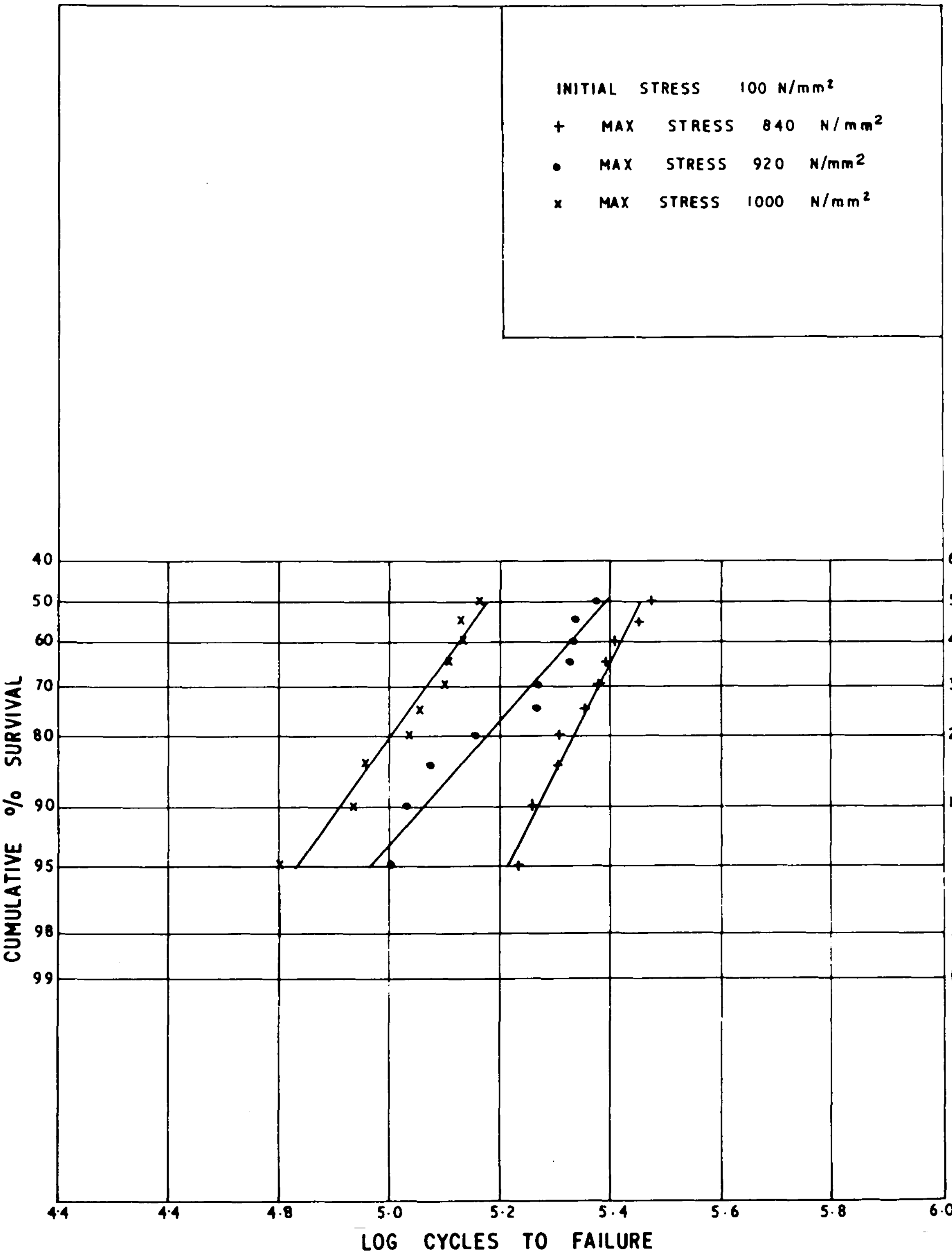

FIG. 6 NORMAL DISTRIBUTION PLOT OF FATIGUE DATA FOR INITIAL STRESS 100 N/mm²

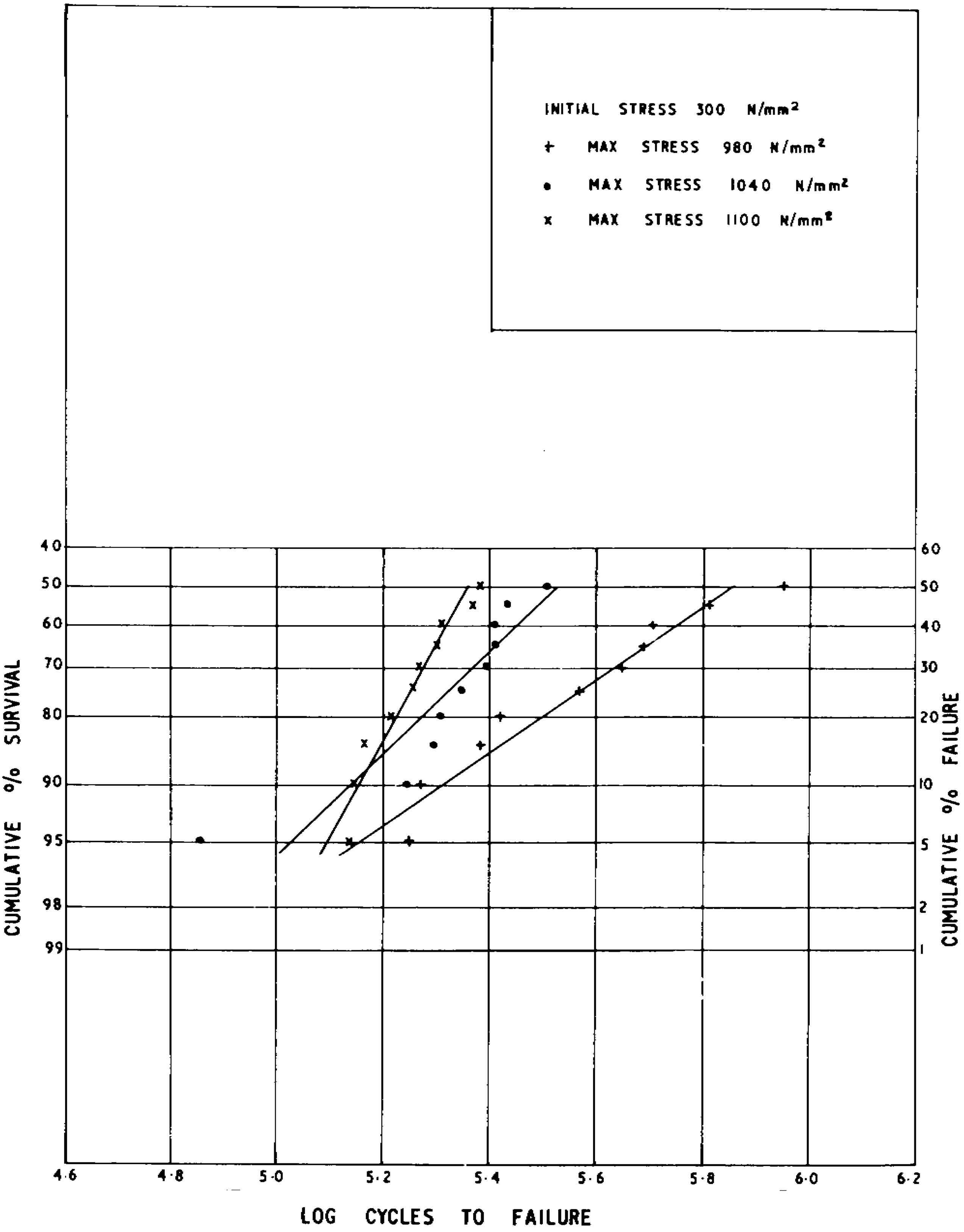

FIG. 7 NORMAL DISTRIBUTION PLOT OF FATIGUE DATA FOR INITIAL STRESS 300 N/mm²

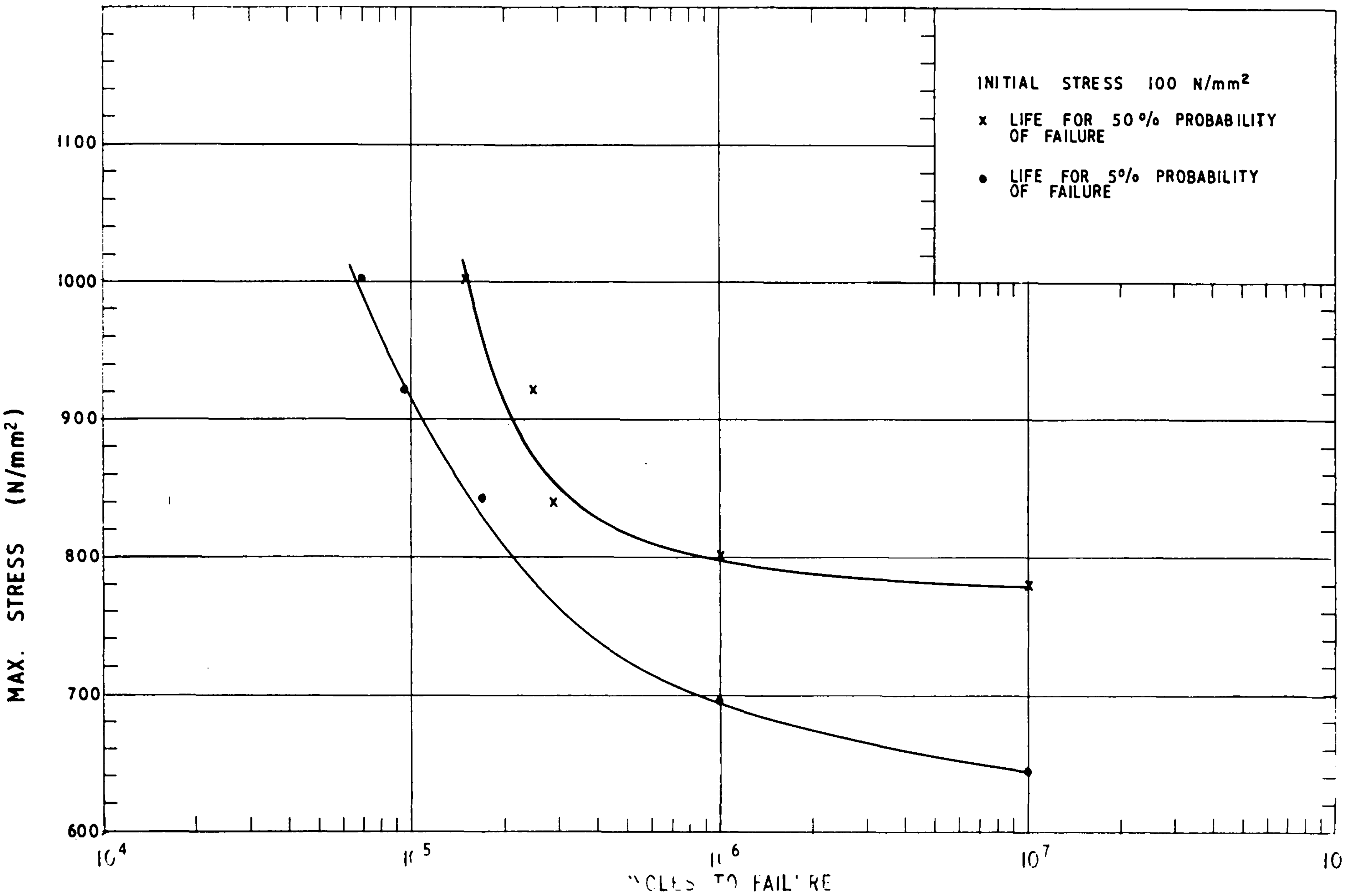

FIG. 8 S/N CURVE PRODUCED BY NORMAL DISTRIBUTION ANALYSIS FOR AN INITIAL STRESS OF 100 N/mm²

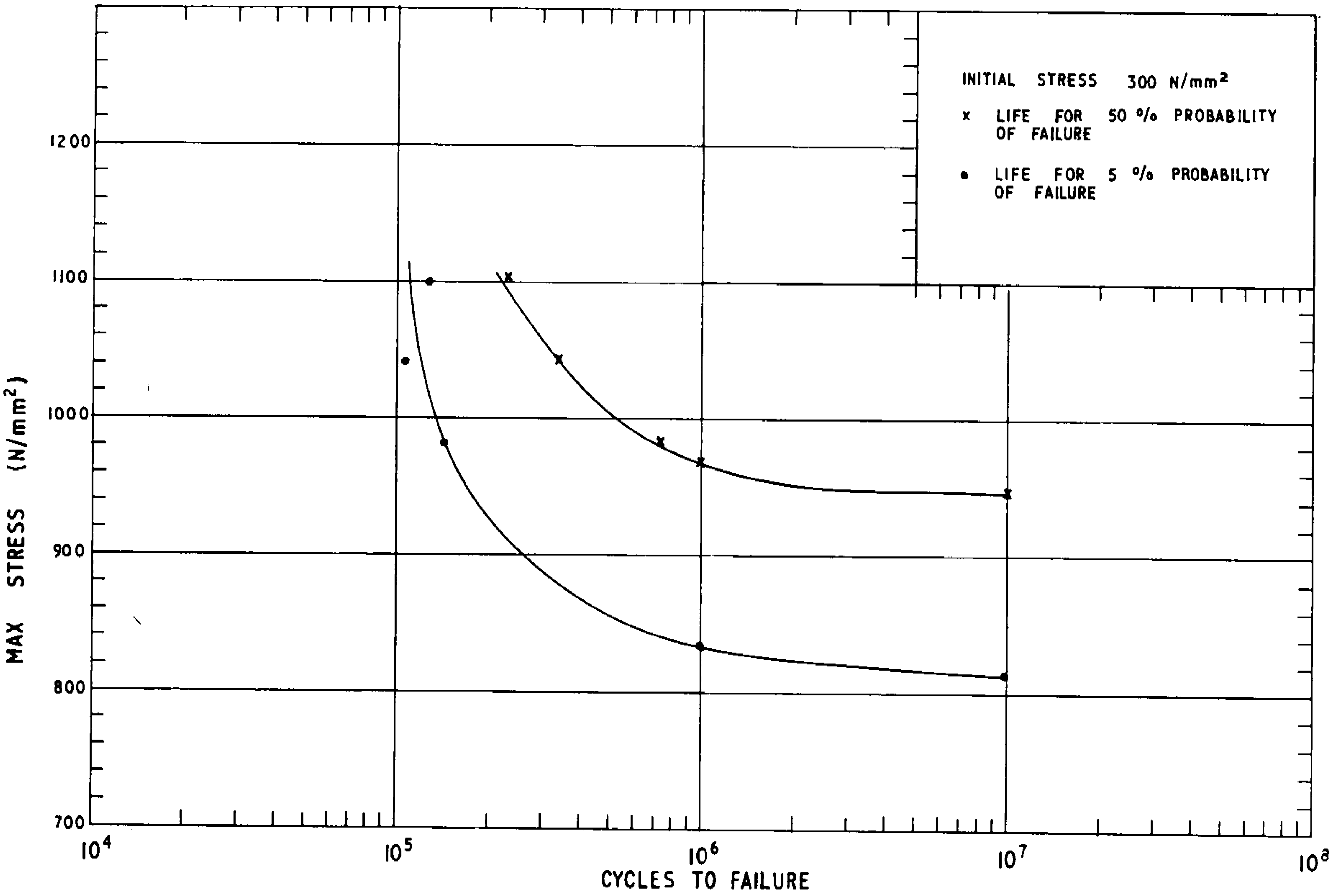

FIG. 9 S/N CURVE PRODUCED BY NORMAL DISTRIBUTION ANALYSIS FOR AN INITIAL STRESS OF 300 N/mm²

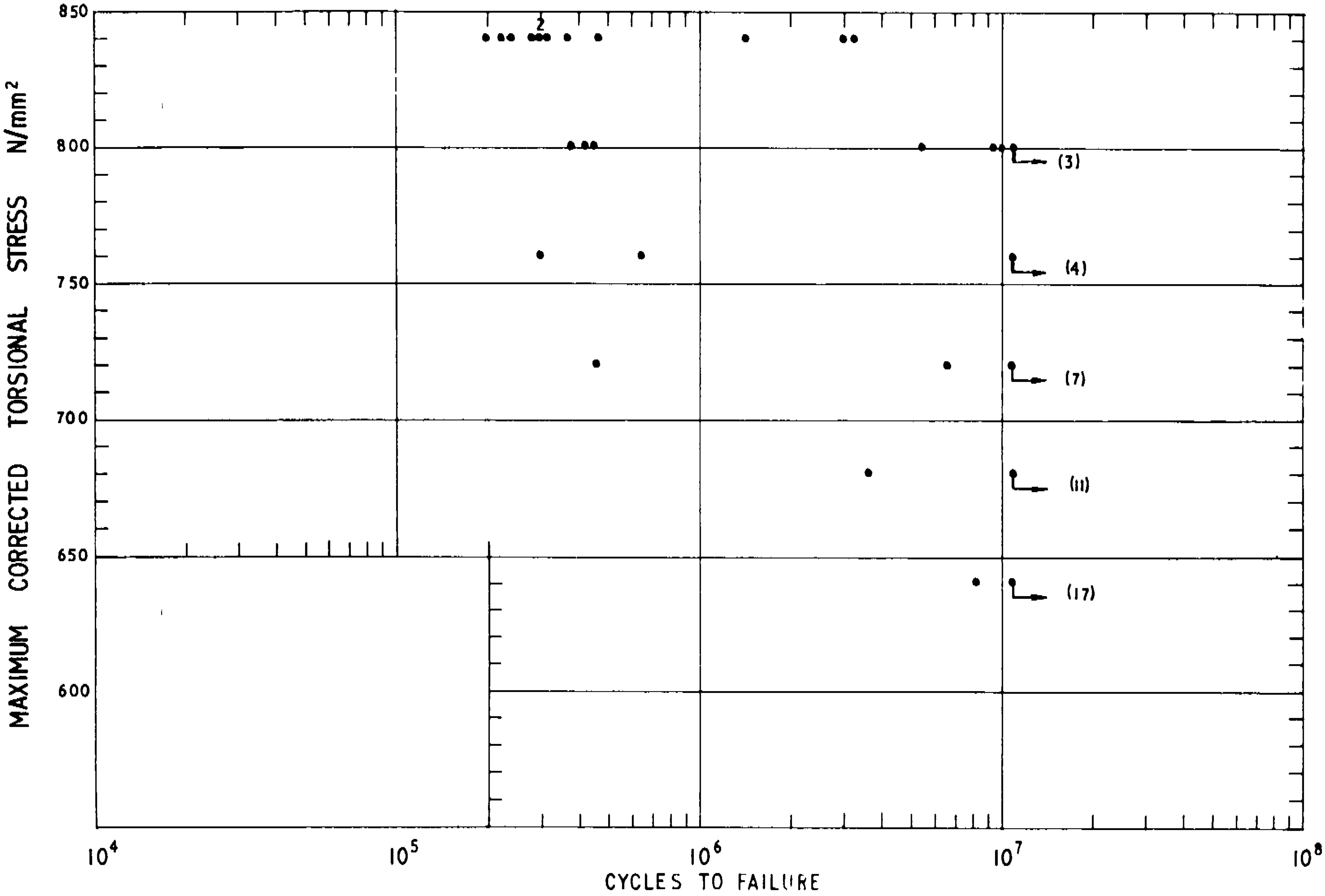

FIG. 10 RESULTS OF PROBIT TESTS FOR INITIAL STRESS OF 100 N/mm²

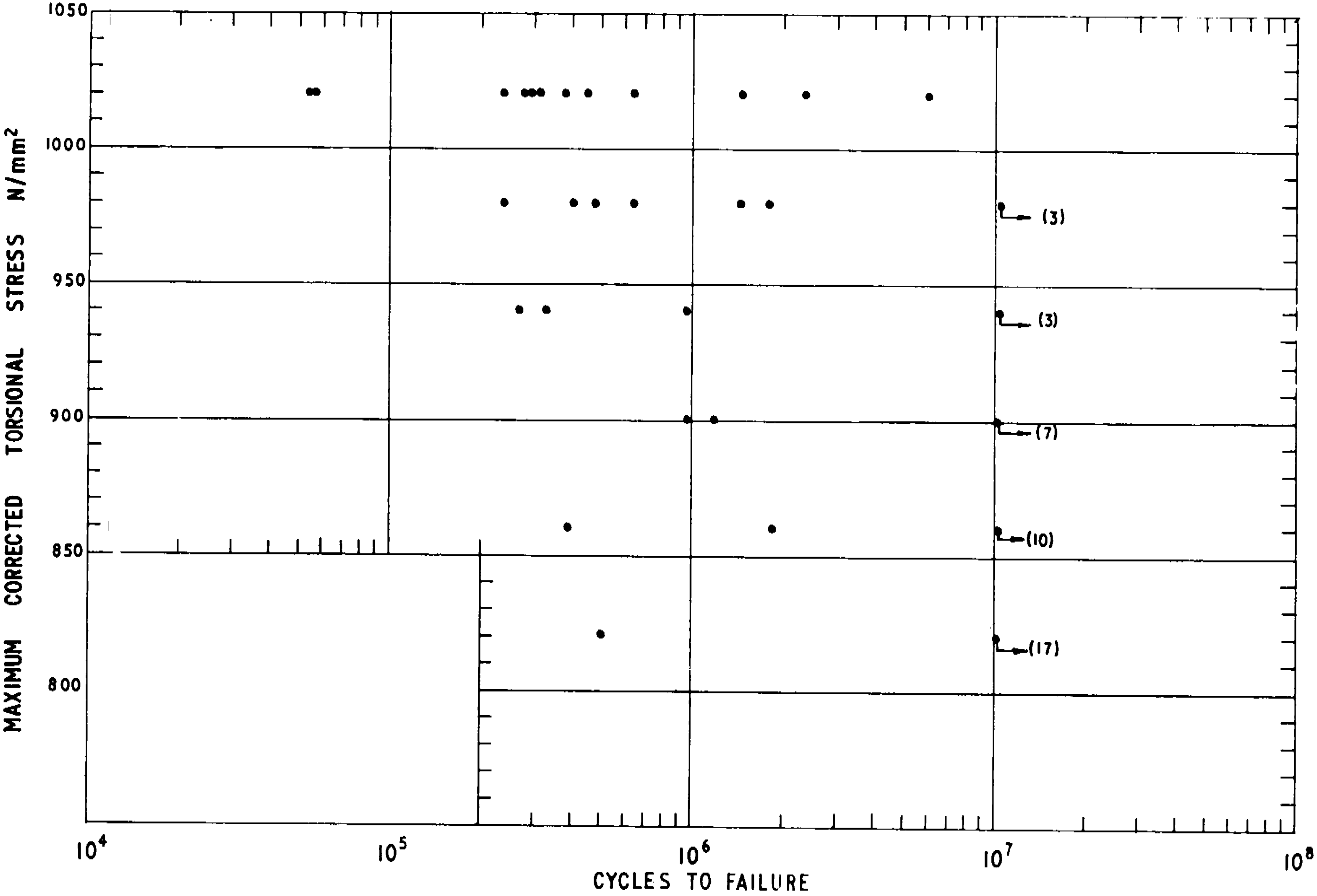

FIG. 11 RESULTS OF PROBIT TESTS FOR INITIAL STRESS 300 N/mm²

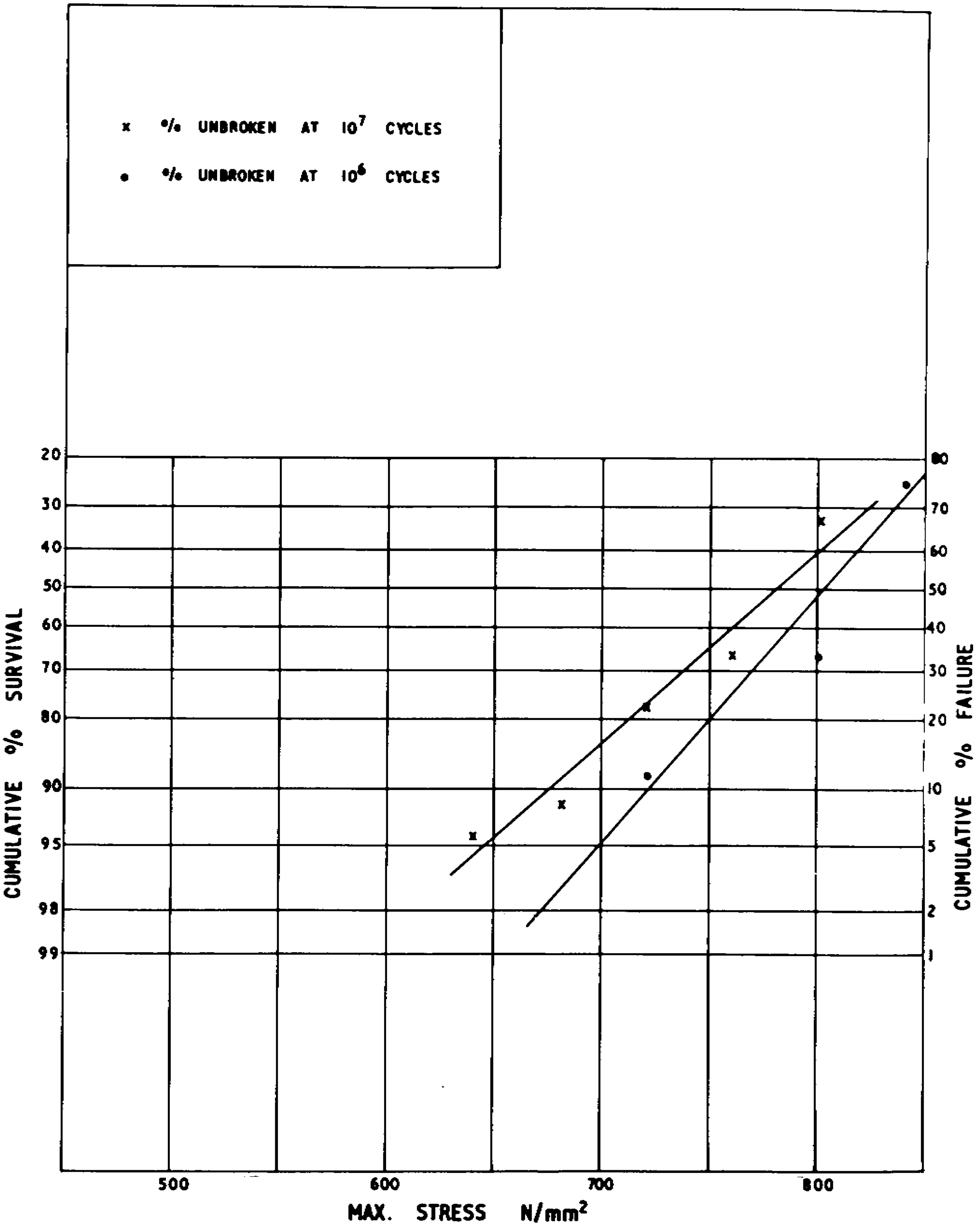

FIG. 12 NORMAL DISTRIBUTION PLOT OF INFINITE LIFE DATA FOR AN INITIAL STRESS OF 100 N/mm²

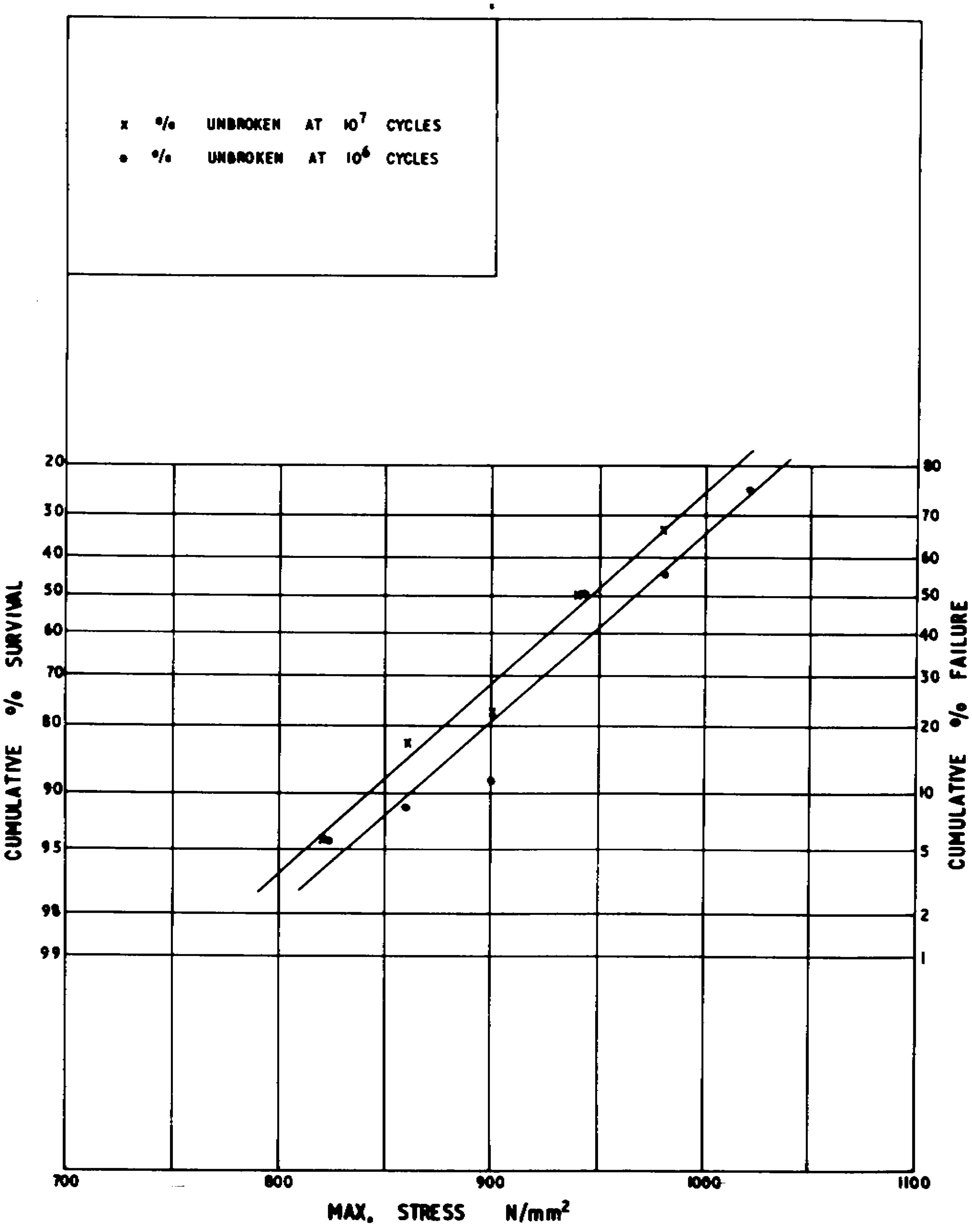

FIG. 13 NORMAL DISTRIBUTION PLOT OF INFINITE LIFE DATA FOR AN INITIAL STRESS OF 300 N/mm²

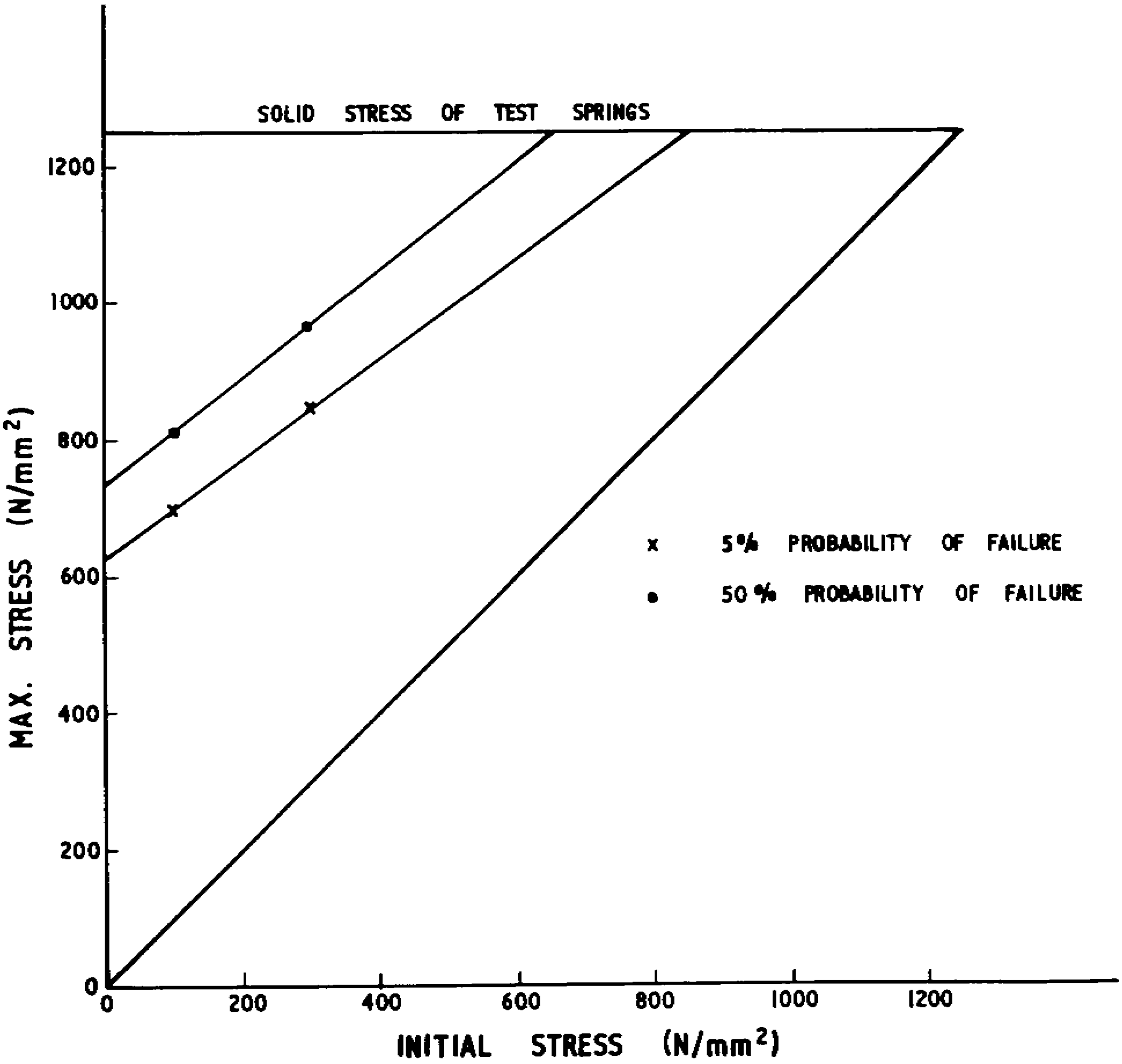

FIG. 14 MODIFIED GOODMAN DIAGRAM FOR 10^6 CYCLES PRODUCED BY PROBIT ANALYSIS.

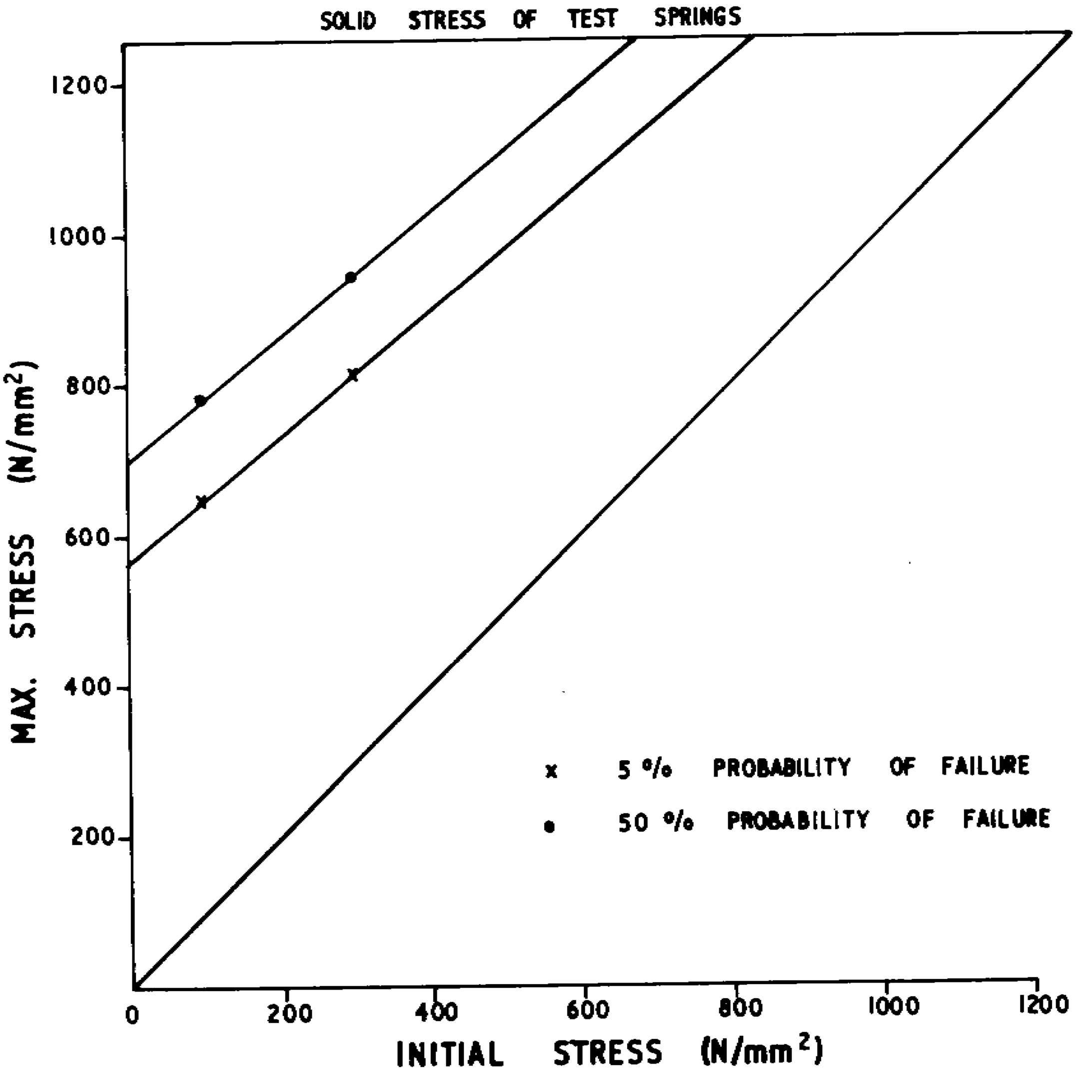

FIG. 15 MODIFIED GOODMAN DIAGRAM FOR 10^7 CYCLES PRODUCED BY PROBIT ANALYSIS.