THE SPRING RESEARCH AND MANUFACTURERS' ASSOCIATION

DYNAMIC STRESSES IN HELICAL COMPRESSION SPRINGS A LITERATURE SURVEY

by

D. Saynor, B.Sc

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SUMMARY

This literature survey was undertaken to bring together in one document the major published information on the dynamic stresses in helical coil springs. The survey considers stresses due to impulsive and periodic loading separately and presents the existing theories for the determination of the stress levels in a spring subject to such types of loading.

Recommendations are made for the use of the more soundly based methods. In most cases these methods are the result of theoretical analysis only and from the literature it appears that little practical work has been directed at verifying the accuracy and applicability of these theories. Suggestions for further work are therefore included which would clarify the situation. Finally, a bibliography is presented which lists articles of a more specialised nature.

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1. INTRODUCTION

When a parallel sided helical compression spring in which the active coils are evenly spaced in the free condition is subjected to a load, it will come to rest in such a position that the active coils will still be equally spaced, although the pitch of the coils will be reduced. Under these conditions the stress in all the active coils will be the same and is dependent only upon the magnitude of the load applied, the spring material used, and spring dimensions.

However, under certain conditions of dynamic loading the coil spacing at localised points in the spring may be much reduced during the loading process. This leads to local stresses in excess of the overall level of stress predicted by the normal static loading formulae. This condition is generally termed 'surging' and occurs in two important loading situations.

The first of these is the application of an impulse or explosive load at one end of the spring which causes a disturbance in the form of a travelling wave to pass down the spring. The wave is reflected at the fixed end and returns up the spring (this is repeated until friction dampens out the wave). The second important dynamic loading situation is the application of a periodic forced motion to one end of the spring which causes a standing wave to be set up in the spring as the periodic disturbances add to one another.

It is seen therefore that in order to be sure that a spring is suitable for a given set of conditions, in addition to the magnitude of the applied load and the spring parameters it is necessary to know the nature of the actuating force (i.e. the time/displacement curve of the moving end). The designer must also know the effect, individual and collective, of each factor on the spring stress.

As there does not seem to be in existance a document which deals completely and succinctly with this problem this survey has been undertaken with a view to discovering such a document or establishing the need for a programme of work to clarify the situation.

2. TYPES OF LOADING

As stated in the introduction, two types of loading are of importance when considering dynamic stresses; these are impulse and periodic loading. It is probably worthwhile emphasising that these two types of loading are very different and require different analysis. The impact condition gives rise to a travelling wave whilst the periodic condition results in the formation of standing waves. This distinction is most clearly made by Gross[1] and should be well understood to avoid confusion. It is not possible to apply the simple formulae derived for impulse conditions in periodic loading situations.

3. NOMENCLATURE

Presented here is a list of symbols with their definitions used in the following sections of this report. The symbols have been unified and may not be those used in the actual article from which an equation is drawn. The units are given in parentheses and are S.I. throughout. Care should be taken if imperial units are used since the acceleration due to gravity may have to be inserted in certain expressions and in others some constants may have a different value. If the S.I. units indicated are adhered to then no such problems will arise.

A	=	cross sectional area of the spring wire	(m^2)
b	=	damping factor	(s^{-1})
b_{min}	=	minimum value of damping factor	(s^{-1})
c	=	spring index	

c_v	=	velocity of the disturbance with respect to the length of the spring wire	(m/s)
C_μ	=	amplitude of the μth sub-harmonic	(m)
d	=	diameter of the spring wire	(m)
D	=	mean coil diameter	(m)
e	=	eccentricity of a circular cam	(m)
f_m	=	maximum acceleration of an impulsive load	(m/s^2)
g	=	acceleration due to gravity (9.81)	(m/s^2)
G	=	modulus of rigidity of the spring material	(N/m^2)
h	=	stroke or cam lift	(m)
k	=	unit rate of spring (SL_o)	(N)
k_1, k_2	=	end coil constants for the natural frequency of the spring	
K	=	curvature correction factor (c+0.2)/(c-1)	
l	=	length of active wire in the spring	(m)
ℓ	=	length of connecting rod	(m)
L_o	=	free length of spring	(m)
m	=	mass per unit length of spring	(kg/m)
M	=	effective concentrated end mass of a spring/mass system	(kg)
n	=	number of active coils in the spring	
N	=	total number of coils in the spring	
P_a	=	alternating applied load	(N)
P_m	=	mean applied load	(N)
P_v	=	load in spring due to an impulse	(N)
r	=	radius of roller cam follower	(m)
R	=	radius of circular cam or crank circle	(m)
S	=	stiffness of spring	(N/m)
u	=	velocity of disturbance with respect to the spring axis	(m/s)
V_m	=	maximum velocity of an impulsive load	(m/s)

δ_i	=	initial precompression of the spring	(m)
ζ	=	space ratio at the maximum working compression of the spring	
λ	=	cam or crank ratio	
ρ	=	density of the spring material	(kg/m^3)
τ_a	=	alternating stress due to slow periodic loading	(N/m^2)
τ_c	=	solid stress of the spring	(N/m^2)
τ_d	=	alternating stress due to high speed periodic loading	(N/m^2)
τ_f	=	acceleration stress due to impulsive loading	(N/m^2)
τ_i	=	initial stress due to any precompression	(N/m^2)
τ_{max}	=	maximum stress in the spring	(N/m^2)
τ_r	=	stress range under periodic loading	(N/m^2)
τ_s	=	static stress due to compression to the working height	(N/m^2)
τ_v	=	velocity stress due to impulsive loading	(N/m^2)
$\bar{\tau}$	=	mean stress in the spring under periodic loading	(N/m^2)
τ_μ	=	increased stress due to resonance of the μth sub-harmonic	(N/m^2)
ω	=	frequency of the exciting motion	(rad/s)
ω_n	=	natural frequency of the spring	(rad/s)
ω_μ	=	critical speed of the μth sub-harmonic	(rad/s)

4. IMPULSE LOADING

Most work on this type of loading has been carried out with gun springs in mind where the impulse loads on the return springs are very great. Impulse loading is limited by the condition that the loading must have ceased before the stress wave has travelled up and down the spring once. If this situation is not met the returning stress wave interacts with the continuing load application to form a complex stress situation within the spring which is an intermediate condition between impulse and periodic loading.

When considering impulse loading it is convenient to think of four stress levels due to four components. These are:-

1. The initial stress in the spring due to initial loading (τ_i).

2. The static stress which would exist if the spring were compressed slowly through the maximum working deflection (τ_s).

3. The stress due to the velocity of the impulse. The so-called 'velocity stress' (τ_v).

4. The stress component due to the acceleration of the impulse (τ_f).

The first and second of the above components are obvious and well known, being derived from the normal static equations. However the third and fourth components are the dynamic stresses which exist in the spring due to the time/displacement function of the impulse.

Several papers have considered the velocity stress (τ_v) due to impulse loading. The most important amongst these are those of Wahl(2), the Ministry of Supply(3), Dick(4), Maier(5), Gross(6) and Warren(7).

Wahl(2) gives the velocity stress as:-

$$\tau_v = \sqrt{2\rho G} \, . \, V_m \qquad \text{..........(1)}$$

For steel springs, the above formulae simplifies to

$$\tau_v = 35.25 \, V_m \qquad \text{..........(1a)}$$

Equations (1) and (1a) are subject to very great assumptions and simplifications which render them useless in almost all cases. The stress is uncorrected, the spring is assumed to be very long (i.e. the wave is damped out before reaching the fixed end and therefore no reflection occurs at the fixed end), and the spring is assumed to be of "normal" design (i.e. the helix angle ≈ 8°).

The M.O.S.[3] improves the above formula due to Wahl, removing one of the simplifications by employing a curvature correction factor (K). The velocity stress is given as:-

$$\tau_v = K \sqrt{2G\rho} \cdot V_m \qquad \text{..........(1b)}$$

Dick[4] and Maier[5] both agree with Wahl for the uncorrected stress, but both remove another simplification by stating that at the end turn, when reflection occurs, the velocity stress will be increased by a factor of two, i.e.

$$\tau_v = 2 \sqrt{2G\rho} \cdot V_m \qquad \text{..........(1c)}$$

Gross[6] and Warren[7] both give formulae for the load in the spring (P_v) due to impulse loading. Gross[6] gives the following

$$P_v = \frac{2S1V_m}{c_v} \qquad \text{..........(2a)}$$

where $c_v = \sqrt{\frac{S1}{\rho A}}$ ....(2b)

and Warren[7] states

$$P_v = 2muV_m \qquad \text{..........(2c)}$$

where $u = \sqrt{\frac{k}{m}}$ ....(2d)

Algebraic manipulation shows (2a) and (2c) to be identical.

Equations (2a) and/or (2c) take into consideration the doubling of the stress at reflection and also allow corrected stresses to be calculated. Springs with helix angles not equal to 8^o can also be accommodated. The formula for velocity stress (τ_v) can be derived from the velocity load (P_v) using the normal relationship as follows .

$$\tau_v = \frac{8cK}{\pi d^2} . P_v \qquad \text{..........(3a)}$$

and $$S = \frac{Gd}{8nc^3} \qquad \text{..........(3b)}$$

Applying (3a) and (3b) the following is derived

$$\tau_v = K \sqrt{8G\rho \frac{1}{c\pi dn}} \; V_m \qquad \text{..........(4)}$$

According to the M.O.S.[3], the velocity stress τ_v should be added to the initial stress (τ_i) and the static stress (τ_s) which would exist if the spring were compressed slowly to the working height.

i.e. $$\tau_{max} = \tau_i + \tau_s + \tau_v \qquad \text{..........(5)}$$

However, the papers by Gross[6] and Warren[7] show this to be in error, and in fact the velocity stress derived by their formulae include the term τ_s. Hence the maximum stress is given by

$$\tau_{max} = \tau_i + \tau_v \qquad \text{..........(6)}$$

Where τ_v is determined from equation (4) above.

For the case where the load has not ceased being applied before the stress wave has travelled down and back up the spring, the situation is quite complex due to interaction between reflected waves. However, an analysis can be made which is given by Gross[6]. The procedure is lengthy, however, and if serious use is to be made of this the original text must be referred to.

Usually τ_{max} is limited by the solid stress in the spring (i.e. the stress in the spring at closure of all coils τ_c). However, the M.O.S.[3] also makes reference to an 'acceleration' stress (τ_f). This stress exists because of the inertia of the coils of the spring to the rapid movement of the impulse load. A paper by Cox[8] substantiates the existence of this component of stress, and shows that it can result in stresses greater than the solid stress. The acceleration stress is given as

$$\tau_f = 29.64\rho DcK\left\{\frac{f_m}{g}\right\} \qquad \text{...........(7)}$$

This is believed to be an empirical relationship which is intentionally conservative for use in the design of gun springs.

It is suggested that this term should be added to the initial and velocity stresses

$$\text{i.e.} \quad \tau_{max} = \tau_i + \tau_v + \tau_f \text{ for } (\tau_i + \tau_v < \tau_c) \qquad \text{...........(8a)}$$

$$\text{or} \quad \tau_{max} = \tau_c + \tau_f \text{ for } (\tau_i + \tau_v \geqslant \tau_c) \qquad \text{...........(8b)}$$

When the velocity (and hence velocity stress) is at its maximum then the acceleration (and hence acceleration stress) is zero. There would seem therefore no reason why an acceleration stress term should be simply added to a velocity stress term. No clarification on this point could be found in the available literature and so it is suggested that for safe design, equations (8a) or (8b) should be used.

5. PERIODIC LOADING

5.1 Simple Harmonic Motion

The vast majority of work undertaken to analyse dynamic stresses in periodic (or cyclic) loading situations has been conducted with valve springs in mind. The periodic loading in this application is relatively complicated, but as a first approximation to simplify the analysis it is assumed that the spring is excited by a purely simple harmonic motion.

The stress levels in a spring subject to high speed periodic loading are shown in Fig. 1 below and compared with the normal low speed stress levels.

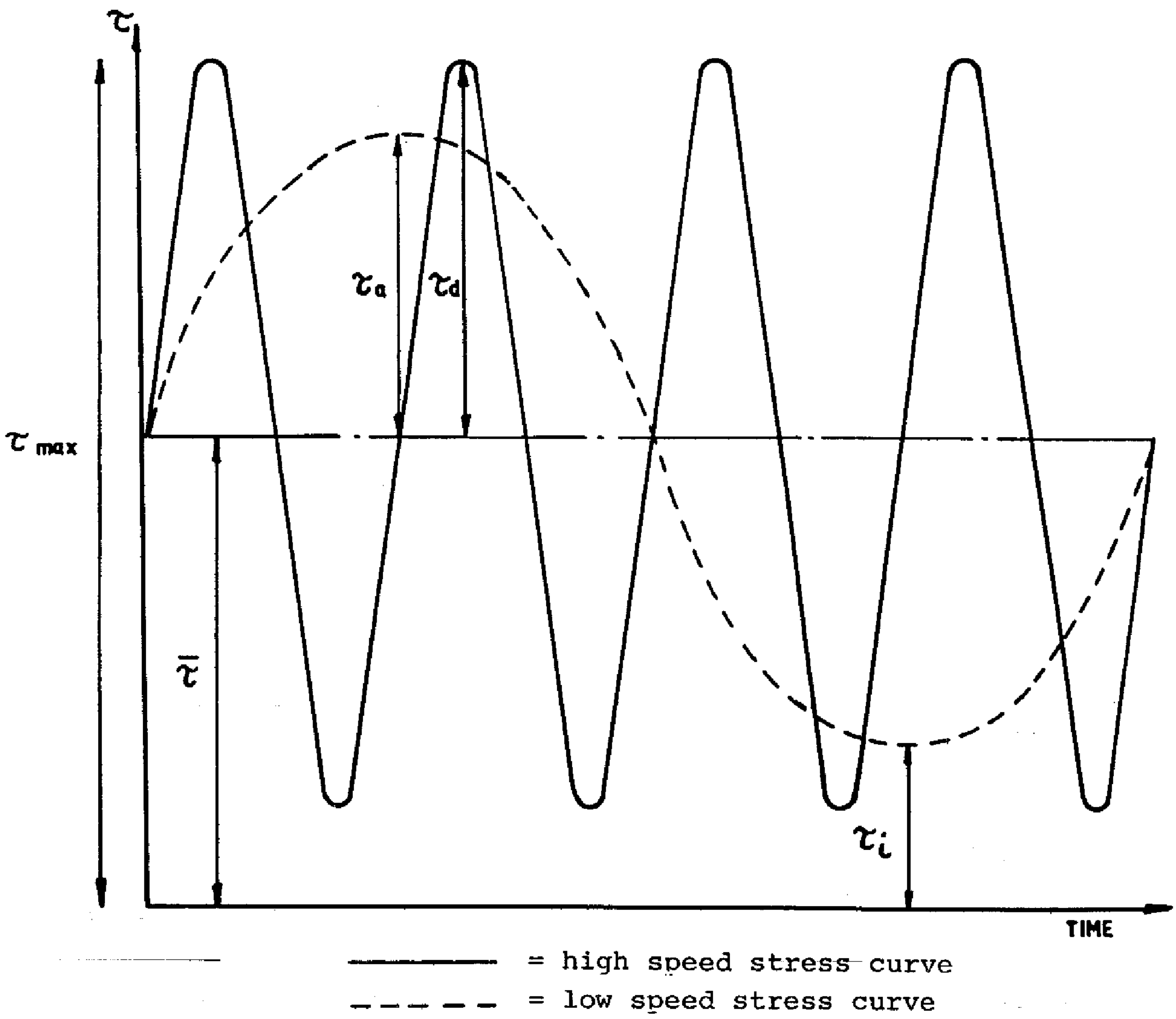

FIG. 1

The maximum stress (τ_{max}) for the high speed cycling is therefore given by:-

$$\tau_{max} = \bar{\tau} + \tau_d \qquad \text{..........(9a)}$$

(cf $\bar{\tau} + \tau_a$ for low speed)

and the stress range is given by

$$\tau_r = 2\tau_d \qquad \text{..........(9b)}$$

(cf $2\tau_a$ for low speed)

It is worth noting that the minimum stress is reduced below τ_i as the speed increases. This is noted in a paper by Edwards(9). If the initial stress is low enough it is possible to generate tensile stress in the compression spring. Edwards(9) also suggests that it is possible to generate stresses in excess of the solid stress (τ_c), but no examples or justification is given for this statement.

To determine the value of τ_d for equations (9a) and (9b) two relationships are given in the literature for different forms of excitation. For a spring excited by a simple harmonic displacement of its free end, Gross(10) solves the following equation of motion.

$$\frac{\partial^2 y}{\partial t^2} = c_v^2 \frac{\partial^2 y}{\partial x^2}$$

and derives the relationship

$$\tau_d/\tau_a = \frac{\pi(\omega/\omega_n)}{\sin\left\{\pi(\omega/\omega_n)\right\}} \qquad \text{..........(10a)}$$

For a spring/mass system excited by a simple harmonic alternating load applied to the free end, Kuran[11] solves the equation of motion for the system

$$M \frac{d^2y}{dt^2} + Sy = P_m + P_a \sin \omega t$$

and derives the relationship

$$\tau_d / \tau_a = \frac{1}{1-(\omega/\omega_n)} \qquad \text{..........(10b)}$$

Clearly, most applications are forced motions of the type analysed by Gross[10], but the designer should be aware of the different approach necessary for spring/mass systems excited by a periodic force covered by Kuran's[11] relationship.

Both equations neglect damping in the analysis and hence the dynamic stresses calculated from the above formulae will be too high. The degree of error will be determined by the actual amount of damping in the system and by the value of ω/ω_n. As resonance is approached (i.e. $\omega = \omega_n$) damping has a relatively greater effect on limiting the actual stresses.

5.2. Complex Harmonic Motion

Equation (10a) due to Gross does not give the complete picture for periodic loading. In situations where the exiting motion is not purely simple harmonic, surging occurs at critical speeds which are sub-multiples of the natural frequency of the spring. For example if a spring with a natural frequency of 10,000 cycles/sec. is excited at speeds of 5,000; 2,500; 2,000 ..... cycles/sec then surging will occur at these critical speeds leading to stresses in the spring much greater than predicted by equation (10a).

The earliest work on this phenomena was by Jehle and Spiller[12], Swan and Savage[13] and Donkin and Clarke[14]. These reports describe the surging effect at these critical speeds and establish its cause and occurrence. However, they do not analyse the stress distribution in the spring, and therefore do not quantify the effect. All were agreed that surging of valve springs at these critical speeds which were sub-harmonics of the natural frequency was due to the displacement curve of the valve cam cycle not being purely simple harmonic. To verify this Jehle and Spiller(13) produced a special cam/follower system which did impart a purely simple harmonic motion to a valve spring and no surging of the spring was noted at speeds below the natural frequency of the spring.

The first known attempt to mathematically analyse the stress distribution in springs during surging under periodic loading conditions which were not purely simple harmonic was made by Hussman(15). He conducted a harmonic analysis of the cam cycle which broke the displacement curve into a Fourier Series of the form.

$$y = C_o + \sum_{\mu = 1}^{\mu = \infty} C_\mu \sin \mu\omega t$$

$\mu = 1, 2, 3, \ldots\ldots\ldots$

He concluded that if $\mu.\omega_\mu = \omega_n$ (i.e. the cam was run at the μth sub-harmonic speed) then resonance of the μth term of the series would predominate and the engine speed ω_μ was a critical speed.

The analysis Hussman performed is not made very clear in his report but is perfectly sound. The E.S.D.U. Data Sheet(16) summarises the results of Hussman's work and presents them in a much more useful form.

The resulting stress/load curve for a spring is shown in Fig. 2 below and illustrates the general characteristics of increasing stress with increasing speed with marked peaks at the critical speeds.

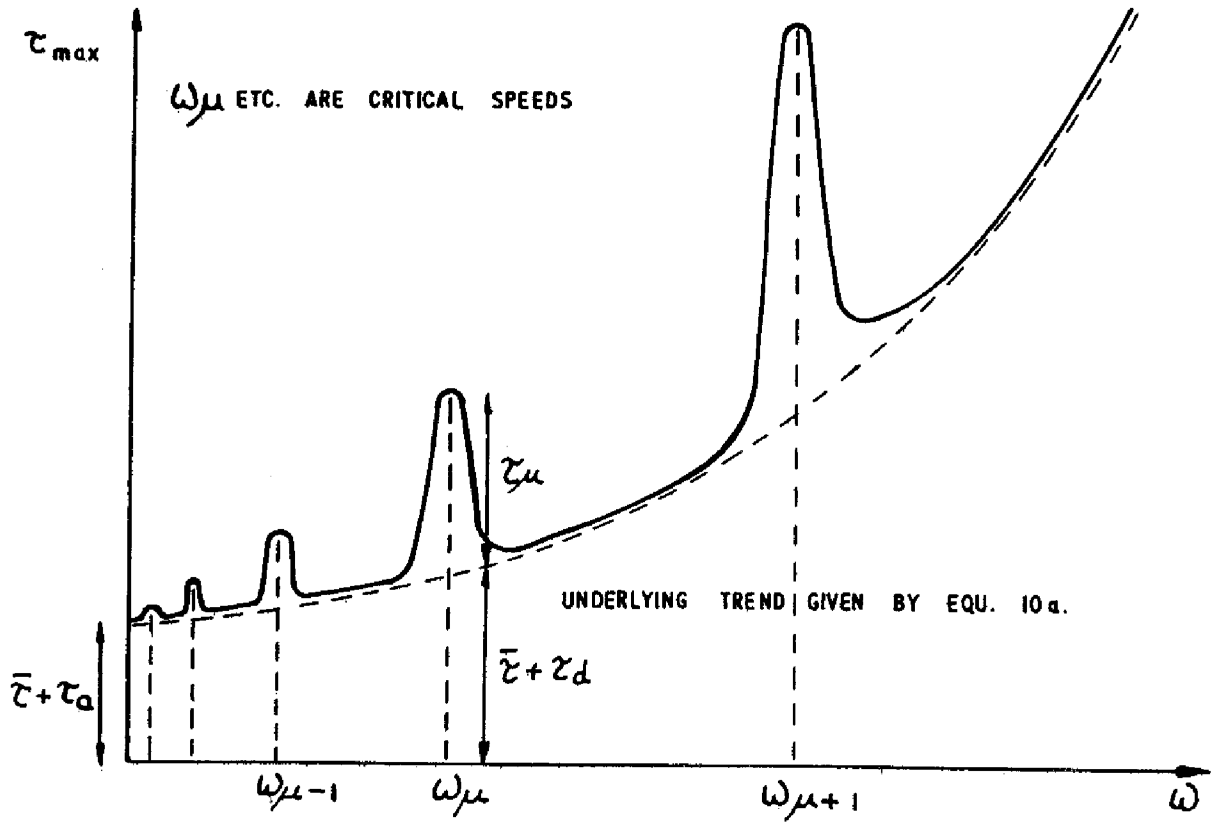

FIG. 2

The value of the additional stress (τ_μ) due to resonance of the μth sub-harmonic at the critical speed ω_μ is given by

$$\tau_\mu = \frac{\omega_n . C_\mu . G.K}{b.\pi c^2 . n.d} \qquad \text{...........(11a)}$$

The maximum stress in the spring is then given by

$$\tau_{max} = \bar{\tau} + \tau_d + \tau_\mu \qquad \text{...........(11b)}$$

and the stress range by

$$\tau_r = 2(\tau_d + \tau_\mu) \qquad \text{...........(11c)}$$

To determine C_μ a harmonic analysis of the cam-lift cycle must be carried out. This is beyond the scope of this report, but papers by Jehle and Spiller[12] and Lee[17] show how this is done giving practical worked examples. The damping factor (b) is an experimentally determined factor which is related to the friction present in the system.

From his analysis Hussman was able to show that harmful surging effects could be minimised by modifying the cam profile. Several examples of this are included in his report, other published designs of low vibration cam profiles are given by Dudley[18] and Bishop[19]. Hussman also investigated common values of the damping factor for normal spring designs. He investigated the effect of initial compression on the value of the damping factor and found that as the initial compression was increased the damping factor attained a minimum value of (b_{min}). From equation (11a) it can be seen that τ_μ is inversely proportional to b, hence the value b_{min} will give the highest value of τ_μ (i.e. this is the worst case). E.S.D.U. 65005 gives the value of b_{min} in a graphical manner as a function of the amplitude of the harmonic (C_μ).

5.2.1 Damping

Hussmans work represented a great step forward when it was published. However, with respect to the damping factor, the method was very crude. This was not surprising since the analysis of damping is very complex. In the initial equation of motion, damping is assumed to be proportional to velocity. This is a gross simplification and although it simplifies the mathematical analysis it leads to problems when assigning values to the damping factor for practical design. Extensive work by Kras[20] attempting to determine analytical relationships between initial compression and damping factor only appeared to underline this difficulty. However, work by Kushiyama and Ayabe[21] does give a more structured approach to the problem although again wide scatter in results is apparent.

They recommend the following values for the minimum damping factor b_{min} which they say produced a successful design of exhaust valve spring for the Mitsubishi U.E. engine.

$$b_{min} = 0.015.\frac{\omega_n}{2\pi} \quad \text{for } \zeta < 0.003 \qquad \text{..........(12a)}$$

$$b_{min} = 1.6(\zeta)^{0.8}.\frac{\omega_n}{2\pi} \quad \text{for } \zeta > 0.003 \qquad \text{..........(12b)}$$

Where ζ is the space ratio for the uth sub-harmonic at the maximum working compression and is given by

$$\zeta = \frac{C_\mu}{L_o-(\delta_i + h + N.d)} \qquad \text{..........(12c)}$$

The above relationships are extremely useful allowing the damping factor to be estimated by calculation rather than by interpolation from empirically derived curves. However, the values obtained for b_{min} from equations (12a) or (12b) and (12c) should be treated with caution since the relationships were empirically derived from tests on only a limited number of springs. It is clear that estimation of the damping constant is the greatest source of error in determining τ_μ since very little data is published for a wide range of springs.

5.2.2 Natural Frequency

Another difficulty which exists when attempting to calculate the stresses due to surging in periodic loading is that of the determination of the natural frequency of the spring. Hussman[15], Kras[20] and Marti[22] all found that the natural frequency of a spring was not a constant. As with damping, natural frequency is a function of the initial compression of the spring and the end coil formation. All three investigated the effect, but only Hussman attempted to give an analytical relationship:-

$$\omega_n = \frac{(1 + k_2\ (^{\tau}i/\tau_c))}{cD(n + k_1)} \sqrt{\frac{G}{2\rho}} \qquad \text{..........(13)}$$

k_1 and k_2 are constants which were experimentally determined from measurements on 13 springs. The scatter on these constants was quite broad and the following values given by Hussman for k_1 and k_2 are mean values.

$k_1 = 0.83$ and $k_2 = 0.185$ for closed ground ends

$k_1 = 0.4$ and $k_2 = 0.1$ for open ends ground

Kras[20] points out that this is far from accurate, but tests performed by him could give no better approximation than that presented by Hussman.

5.2.3 Other Difficulties

Problems in analysis which affect the determination of the value of C_μ but which are beyond the scope of this report are related to the elasticity of the valve train, and tappet (or valve) clearance. The former has been investigated by Dudley[18], Bishop[19], Olmstead and Taylor[23] and Turkish[24] amongst others. The problem of tappet clearance has been examined by Swan and Savage[13] and Hussman[15].

An article by Schlaefke mentioned but not referenced in a report by Kras[20] also considered wear of the spring and manufacturing errors which over order numbers of 8 to 18 increased each value of C_μ by approximately 0.005 mm.

5.2.4 Circular Cam or Crank Drives

The phenomena of surging at speeds which are sub-multiples of the natural frequency of the spring extends to both circular cam and crank drives since neither of these impart a purely simple harmonic motion to the spring. These 2 types of drive are considered to be of importance because of their common use in forced motion fatigue testing machines.

The analysis of this situation is identical to that performed on engine valve springs. However, there is no need to resort to a harmonic analysis to determine the values of each C_μ. Instead, each value can be calculated from a knowledge of the term λ, where for a crank drive

$$\lambda = \frac{R}{\ell} \qquad \text{..........(14a)}$$

and for a circular cam drive

$$\lambda = \frac{e}{R + r} \qquad \text{..........(14b)}$$

The method of analysis is given in detail by Gross[1 and 6].

6. DISCUSSION

Neglect of the dynamic stress has the most detrimental effect in periodic loading situations. This is because in an impulse loading application the spring is usually designed to be safe at its solid stress; and if this is the case then the spring will not fail due to the dynamic stresses unless the accelleration is exceptionally great.

In periodic situations, however, the increased maximum stress and stress range brought about by dynamic effects both seriously reduce the fatigue life of the spring. Also in situations where a spring is run at a critical speed this stress range is cycled through at a rate equal to the natural frequency of the spring, again reducing the life time of the spring drastically.

To reduce these effects designers can adopt several approaches, the following are a summary of methods mentioned in the literature.

1. Springs are designed with a very high natural frequency. This reduces the values of C_μ for the relevant critical speeds.

2. The cam load/lift cycle can be modified. This reduces the relevant values of C_μ.

3. Variable rate springs can be used (e.g. variable pitch or variable diameter springs). This causes a continuous variation of the natural frequency of the spring as it is compressed so that the spring is thrown out of surge.

4. Spring dampers can be fitted. This causes an increase in friction and increases the value of the damping in the system.

6. CONCLUSIONS

Theories and design methods have been found and summarised which cover almost all aspects of dynamic stressing. The methods recommended for use in this report are the ones which appear to be most soundly based, but these must be put to the test to evaluate their accuracy in actual dynamic situations.

For impulsive situations the following is recommended for the calculation of the velocity stress component.

$$\tau_v = K\sqrt{8G\rho \cdot \frac{1}{\pi c d n}} \cdot V_m$$

The acceleration stress should be determined using

$$\tau_a = 29.64\rho \; cKD\left\{\frac{f_m}{g}\right\}$$

For situations of high speed loading which are not rapid enough to be defined as impulse loading then the method outlined by Gross[4] is recommended.

For periodic loading applications subject to simple harmonic motion of the displaced end then the following is recommended

$$\tau_d = \tau_a\left\{\frac{\pi(^{\omega}/\omega_n)}{\sin\{\pi(^{\omega}/\omega_n)\}}\right\}$$

For spring/mass systems excited by a simple harmonic periodic force then

$$\tau_d = \tau_a \left\{ \frac{1}{1 - {}^{\omega}/\omega_n} \right\}$$

For complex harmonic loading the increase in stress produced by resonance at a critical speed should be determined using

$$\tau_\mu = \frac{\omega_n . C_\mu . G.K.}{b_{min} . \pi c^2 . nd}$$

In this equation the natural frequency should be determined using equation (13) and the minimum damping factor should either be read from Fig. 8 of the E.S.D.U. 65005 data sheet or using equations (12a, b and c).

8 SUGGESTIONS FOR FUTURE WORK

From the findings of this literature survey it is clear that a full programme of work needs to be conducted to investigate practically the question of dynamic stresses in springs. As stated in the discussion, this is more important with regard to periodic loading situations where according to theory the dynamic stress effects should result in premature failure.

As a first stage the existence of increased maximum stress and stress range due to high speed periodic loading must be verified qualitatively since the effect is only a theoretical one derived from mathematical analysis. This could best be done by a comparison of the fatigue lives of springs run at high and low speeds. As well as indicating the existence of the dynamic stresses this work would also indicate the reduction in life which can be expected as a result of the high speed application and show whether or not it is a significant factor.

If the first stage of work provides positive evidence that dynamic stresses do exist and significantly reduce fatigue life, then a second stage should be undertaken to investigate the actual stress levels in springs under high speed loading. The existing equations and design methods could then be verified for relevance and accuracy, and the fatigue life of any general spring predicted for any speeds of operation other than critical speeds.

Various possible methods exist for determining the stress levels in springs, but probably the only method which is practical with present technology which could be applied to high speed applications is to attach strain gauges to the spring. For both the first and second stages of work a high speed machine for testing springs will need to be designed and developed.

A third stage of work which is suggested here is to investigate and verify the stress τ_μ introduced by resonance at critical speeds. As a forerunner to this work the equations and data on natural frequencies and minimum damping will require verifying and extending if possible since errors in these values lead to errors in the value of τ_μ predicted.

If these suggestions are implemented it will lead to a situation in which the designer can analyse dynamic stresses over the full speed range and predict corresponding fatigue life with a greater degree of confidence than exists at present where it appears that he is relying on theory with little or no practical verification.

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