THE SPRING RESEARCH & MANUFACTURERS ASSOCIATION

SPRING DESIGN

Report No. 392

Reprinted from the Spring Design Series

appearing in 20 monthly parts in SRAMA's Newsletter

June 1984 to February 1986

The following pages have been reproduced without editing or amendment, and so any errors repetitions or inconsistencies have been perpetuated here.

FEBRUARY 1986

CONTENTS

SPRING DESIGN SERIES

Introduction

The majority of springs used in industry are parallel sided compression tension and torsion spring types and this series is intended to be a guide for engineers and spring users to the specification and design of the most widely applied springs.

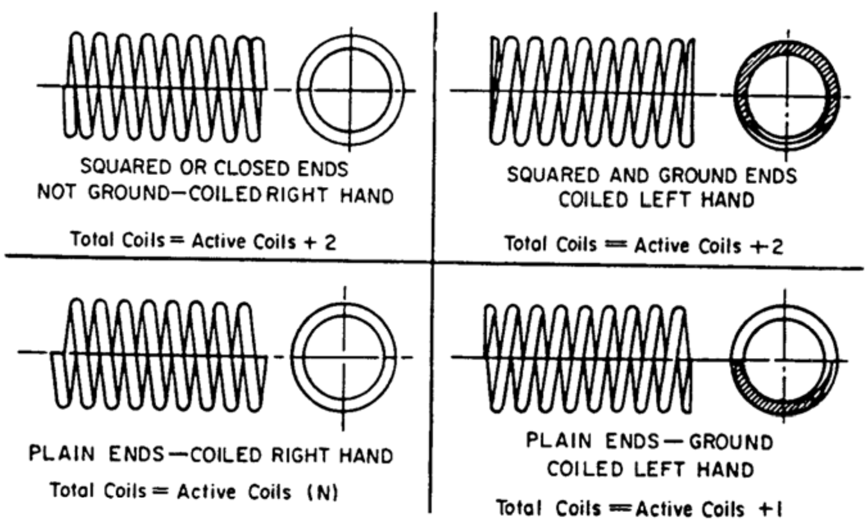
The information given is basic and the design formulae should be well within the scope of anyone with a working knowledge of algebra and an appreciation of the mechanical properties of the materials from which springs are made.

PART 1 COMPRESSION SPRINGS

Compression springs represent around 80% of all springs produced by spring making companies. Where there is a choice of the type of spring to be fitted in a component or mechanism usually the compression spring is preferred in that the stress concentration occurring at the ends of tension springs could result in loss in performance. Also wherever possible the spring should be designed in round wire rather than rectangular or square sections as the former is much cheaper, has a better surface and is easier to obtain.

Design Considerations

Fig. 1 shows the four common end forms used on compression springs.



Types of Ends

The number of active (working) coils is influenced by the end formation. The relationship between total coils (N) and active coils (n) is shown in the table below. In addition the solid length of a compression spring (i.e. the length of the spring when all coils touch each other) is dependant upon the end type. This is also tabulated below.

Type of End	Number of active coils (n)	Solid Length (Ls)
Closed and ground	N - 2	(N-½) x d
Closed	N - 2	(N+1) x d
Open	N	(N+1) x d
Open and ground	N - 1	N x d

d = wire diameter; N = no. of total coils

Spring Index (c)

The ratio, $\frac{\text{mean coil diameter (D)}}{\text{wire diameter (d)}}$ is known as the spring index (c).

For circular section wires c should be not less than 3½ and not more than 12 if the springmaker is to maintain accuracy during coiling.

Spring Rate (s)

This is the change in load per unit deflection, and is measured in lbs/in or Newtons/mm (e.g. if a spring has a rate of 40 lb/in then the load will increase by 40 lbs for every inch through which the spring is deflected). The rate of any spring may be determined by:- a) Deflect the spring to approx 20% of available deflection and measure the load (P1) and spring length (L1), (b) Deflect the spring to not more than 80% of total available deflection and measure load (P2) and spring length (L2), (c) Cabulate Rate (S) from:-

$$S = \frac{(P2 - P1)}{L1 - L2} \text{ lb/in or (N/mm)}$$

In spring design the standard formula for rate is:-

$$S = \frac{Gd^4}{8nD^3}$$

where G = Rigidity Modulus
d = wire diameter
n = active coils
D = mean diameter

Rigidity Modulus (G)

The amount of elastic deflection in a spring depends on two constants called moduli of elasticity which have distinct values for every spring material. The rigidity or shear modulus (G) is used for compression and tension spring calculations because such springs are stressed in torsional shear.

Values for G expressed in kN/mm^2 and lb/in^2 for a range of spring materials are given in the table below:-

Modulus Values for Spring Material

	G (kN/mm²)	G (lbf/in²)
Hard drawn carbon steel	79.3	11.5×10^6
Carbon steel for hardening and tempering	79.3	11.5×10^6
Silicon manganese steel	79.3	11.5×10^6
Chromium vanadium steel	79.3	11.5×10^6
Martensitic stainless steel	79.3	11.5×10^6
Austenitic stainless steel	65.5-75.8	$9.5 - 11.0 \times 10^6$
Phosphor Bronze	43.0	6.25×10^6
Hard drawn brass wire	36.0	5.25×10^6
Copper beryllium	41.3	6.0×10^6
Monel	65.5	9.5×10^6
Inconel X750	76.0	11.0×10^6
Nimonic 90	82.7	12.0×10^6
Titanium alloys	34.5-41.4	$5-6 \times 10^6$

Design Stresses

When a compression spring is deflected the wire is twisted under torsion and stresses are produced which are higher on the inside of the coils. In calculations this higher stress on the inside is taken into account by multiplying the standard formula for stress by a correction factor (K) which increases as the curvature of the wire increases i.e. lower spring index.

The basic stress formula corrected for curvature is:-

$$\text{Stress } (\tau) = \frac{8PD}{\pi d^3} K$$

where P = axial load

D = mean coil diameter

d = wire diameter

$K = \frac{c + 0.2}{c - 1}$ = curvature correction factor

$c = \frac{D}{d}$ = spring index

Maximum Stresses

Having obtained the stress calculated from the formula the designer needs to know whether it is within the limiting stress values of the material to be used. Fig. 2 gives the maximum design stresses which are recommended for commercial springs made from a number of readily available spring materials. In general, compression springs should be designed so that the stress at solid is within the limits shown on the curves. A further restraint is that the spring should be operated so that no more than 85% of the total available deflection is used. This ensures that the spring does not go solid under normal operating conditions.

NOTE The design stresses in Fig. 2 are for springs which have been prestressed by being compressed to solid a number of times (usually 3) to remove initial set and stabilise the free height of the spring. They are also intended for springs which are statically loaded i.e. held in one position throughout their working life. Springs which do not undergo more than 10,000 (10^4) reversals of load during their working life may also be designed to the same maximum stress levels. Where the loading cycle is repeated more than 10^4 times the effects of fatigue have to be considered and the maximum permissible stresses will be reduced (See information in SRAMA Material Selector).

A further limitation of Fig. 2 is that no account has been taken of relaxation. A compression spring will lose load when stressed to high stress levels for extended periods of time. This effect is more pronounced at high temperatures but can be significant at room temperatures for certain materials. If load loss due to relaxation could affect the spring performance then the maximum stress may need to be limited to lower values than those detailed in Fig 2. (See SRAMA Material Selector for such information).

Design Example

A compression spring has the following specification:-

Outside diameter (D_o) = 25 mm

Free length (L_o) = 80 mm

Load (P) = 267 N at 44 mm compressed height (i.e. 36 mm deflection from free height)

The maximum allowable solid length to allow 15% residual deflection is therefore $\frac{36}{.85}$ = 42.4 mm max.

Ends closed and ground.

Determine wire diameter (d) Stress (S), number of active coils etc.

1. Assume a trial wire diameter of around 1/8 of O.D. say 3.00 mm.

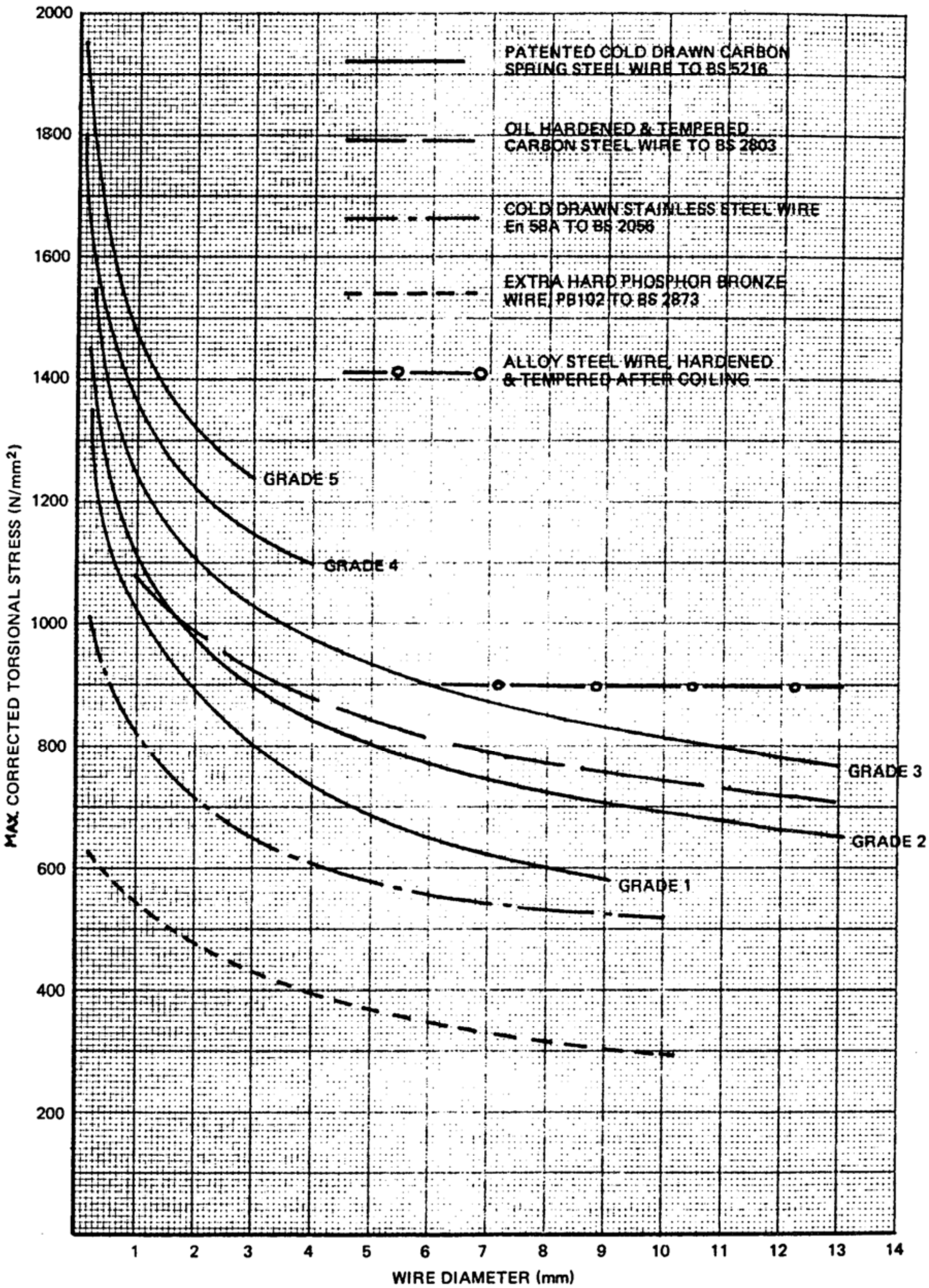


Fig.2 **Relative maximum permissible static operating stresses for spring materials which have been stress relieved and prestressed.**

2. Calculate spring index (c) and stress correction factor (k)

$$C = \frac{D_o - d}{d} = \frac{25 - 3}{3} = 8$$

$$K = \frac{C + .2}{C - 1} = \frac{8 + .2}{8 - 1} = 1.17$$

3. Calculate operating stress from formula:-

$$\tau = \frac{8PD}{\pi d^3} K = \frac{8x\ 267 \times 22 \times 1.7}{\pi \times 3^3}$$
$$= 648\ N/mm^2$$

Therefore from the design stresses (Fig. 2) it is possible that cold drawn carbon steel wire would be suitable. However, the solid stress needs to be calculated before this material can be accepted.

4. Calculate rate as follows:-

$$S = \frac{267}{36} = 7.42\ N/mm$$

5. Calculate number of active coils:

from $n = \frac{Gd^4}{8SD^3} = \frac{79300 \times 3^4}{8 \times 7.42 \times 22^3}$
= 10.16 say 10.1/4 coils
Therefore Total coils (N) = (n + 2) = 12.1/4 coils
and solid length (L_s) = (N - 1/2) x d

= 11.75 x 3
= 35.25 mm

This is less than the calculated maximum solid height of 42.4 mm (max) which is acceptable.

6. Solid load (P_s) = (L_o - L_s) x S
= (80 - 35.25) x 7.42
= 332N

Solid stress (τ_s) = $\frac{8P_sD}{\pi d^3} K$
$= \frac{8 \times 332 \times 22 \times 1.17}{\pi \times 3^3}$
$= 806\ N/mm^2$

This is slightly higher than the design stress indicated in Fig. 2 for grade 1 patented cold drawn wire to BS 5216 but grades 2 or 3 would be acceptable.

Alternatively, if a design in grade 1 were required then the procedure from step 2 needs to be repeated with the next largest wire size (say 3.15)

$$C = \frac{25 - 3.15}{3.15} = 6.94$$

$$K = \frac{6.94 + 0.2}{6.94 - 1} = 1.20$$

$$\tau = \frac{8 \times 267 \times 21.85 \times 1.20}{\pi \times 3.15^3} = 570 \text{ N/mm}^2$$

$$n = \frac{79300 \times 3.15^4}{8 \times 7.42 \times 21.85^3} = 12.6 \text{ (say 12.1/2)}$$

$$L_s = (14.5 - 1/2) \times 3.15 = 44.1\text{mm}$$

This is larger than the maximum of 42.4 mm allowed. Hence, a design using this larger wire size of 3.15 mm is not possible unless the outside diameter could be increased (say to 26 mm) Calculations could then be repeated from step 2.

$$C = \frac{26 - 3.15}{3.15} = 7.25$$

$$K = \frac{7.25 + 0.2}{7.25 - 1} = 1.19$$

$$n = \frac{79300 \times 3.15^4}{8 \times 7.42 \times 24.85^3} = 11.02 \text{ (say 11)}$$

$$L_s = (13 - 1/2) \times 3.15 = 39.4 \text{ (acceptable)}$$
$$P_s = (80 - 39.4) \times 7.42 = 301 \text{ N}$$

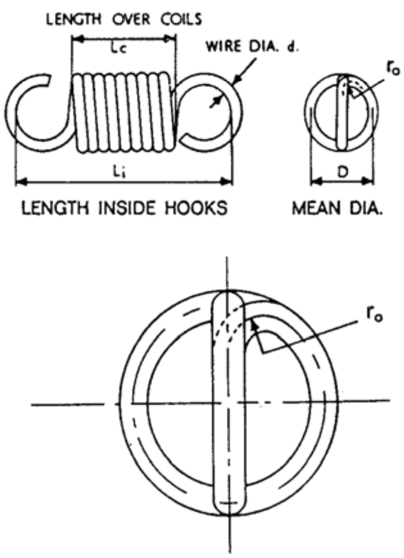
$$\tau_s = \frac{8 \times 301 \times 22.85 \times 1.19}{\pi \times 3.15^3} = 667 \text{ N/mm}^2$$

This stress is well within the allowable for BS 5216 grade 1 material as shown on Fig. 2.

Part 2 Extension Springs

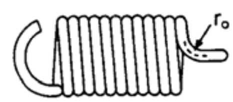
Helical extension springs differ from compression springs only in the method of load application and accordingly in the formation of the spring ends. The application of a tensile load requires either hooks or loops which are part of the spring (fig 1, 2.) itself, or separate component parts with swivel eyes of threaded bolts which are held loosely in curved ends or screwed into ends (fig 3). The latter screwed in inserts do however allow the number of active turns to be varied and hence allow the spring rate to be accurately set.

The major problem with most hook designs is that they incorporate stress raising bends and hence extension springs tend to exhibit poorer fatigue performance than the equivalent compression springs.

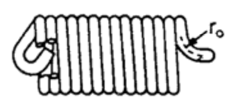


ENLARGEMENT OF END VIEW

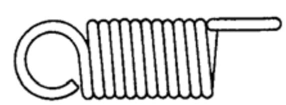
Fig. 1.



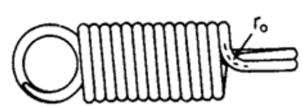
(a.) HALF HOOK OVER CENTRE



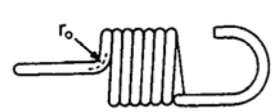
(b.) HALF LOOP OVER CENTRE



(c.) FULL HOOK AT SIDE

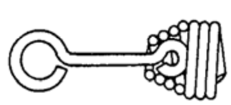


(d.) DOUBLE TWISTED FULL HOOK OVER CENTRE

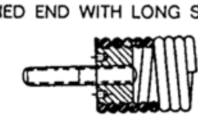


(e.) LONG ROUNDED END HOOK OVER CENTRE

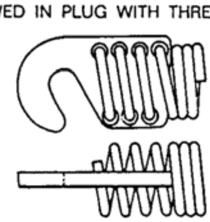
Fig. 2.



(a.) CONED END WITH LONG SWIVEL EYE



(b.) SCREWED IN PLUG WITH THREADED BOLT



(c.) SCREWED IN FLAT HOOK

Fig. 3.

Load-Deflection Characteristic

The load-deflection characteristic of an extension spring differs from that of a compression spring due to the effect of initial tension. The effect of initial tension is to hold the coils in contact and thus to produce any deflection in the spring the applied force must be greater than the initial tension (fig 4).

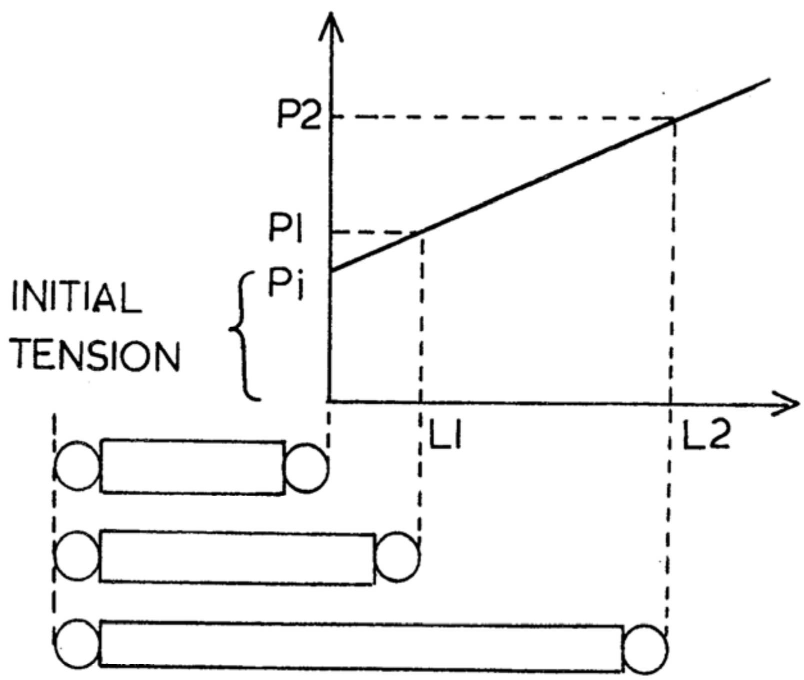


Fig 4

By inspection it can be seen that if $P_i=0$ then the load-deflection characteristic is similar to that of a compression spring.

Design details for Initial Tension

The amount of initial tension that can be coiled into a spring is not limitless. It is constrained by the strength of the wire used, the spring index and the manufacturing process. The graph in fig 5 depicts the maximum commercially available initial tension. Very high or very low initial tension (i.e. outside the preferred range) will cause coiling tolerances to increase due to the difficulties of setting up coiling machines at these extreme limits. Heat treatment and/or prestressing operations after coiling will effectively remove some or all of the initial tension.

Maximum Allowable Working Stresses in Extension Springs

The maximum stress that a spring can withstand without taking a permanent set is dependent on the elastic limit of the material. For example with a patented wire to BS 1408R3 the elastic limit will be in the order of 35% of the UTS in the unheat treated condition and 47% of the UTS after a heat treatment of 250°C for 30 minutes. Often a balance will have to be achieved between heat treatment and hence increase in elastic limit and loss in initial tension. Thus generally speaking extension springs can be stressed to the same levels as unprestressed compression springs. It should be remembered that extension springs do not have a physical stop to inhibit over-stressing and thus a minimum residual working range of15% is important.

If the hook design is such that the bending stress is approximately greater than twice that of the torsional body stress then plastic set can occur in the loops and must be catered for.

Fatigue Properties

Extension springs do not operate as well in fatigue applications as their compression spring counterparts. This is often due to loop designs incorporating stress raisers in the form of tight bends or tooling marks. In comparison the extension spring is some 20% poorer in fatigue than a similarly stressed compression spring. It should also be noted that extension springs cannot be shot peened or prestressed easily and hence need to be lowly stressed in fatigue applications.

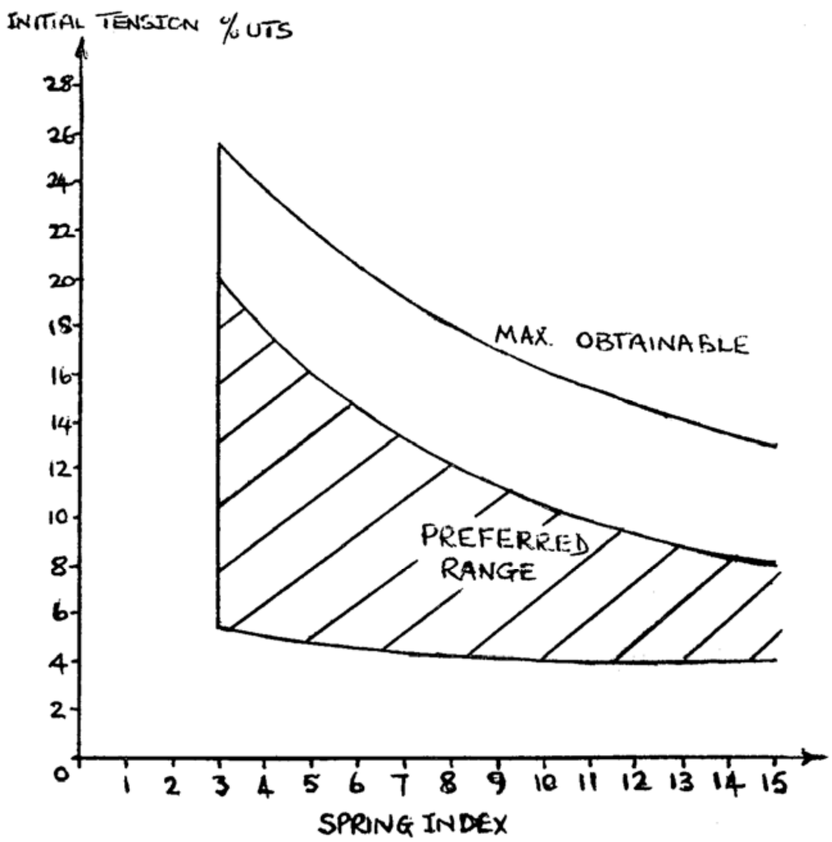


FIG 5 MAXIMUM INITIAL TENSION v. SPRING INDEX

Design Formulae for Extension Springs

Because the normal extension spring is in principle the same as a compression spring the design formulae are the same.

Notation Used

D = Mean coil diameter

d = Wire diameter

c = Spring index D/d

n = No active coils

L_c = Body length

L_o = Free length inside hooks

P = Spring load

P_i = Initial tension load

G = Shear modulus

δ = Extension

S = Spring rate

q = Torsional stress due to P

q_i = Torsional stress due to P_i

K = Sopwith correction factor $\frac{c + 0.2}{c - 1}$

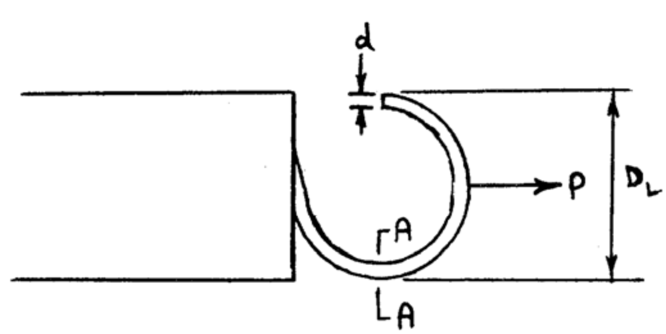
Torsional stress due to P: $q = \frac{8DPK}{\pi d^3}$

Free length: $(N + 1)d + 2$ (hook length)

$= (N + 1)d + 2\ (D-d)$ for normal spring loops

Formulae for Calculating Stress in End Loops

The end loops on a spring are subject to both bending and torsional stress. Experience has shown that failure will usually occur due to the bending stress.



Notation Used

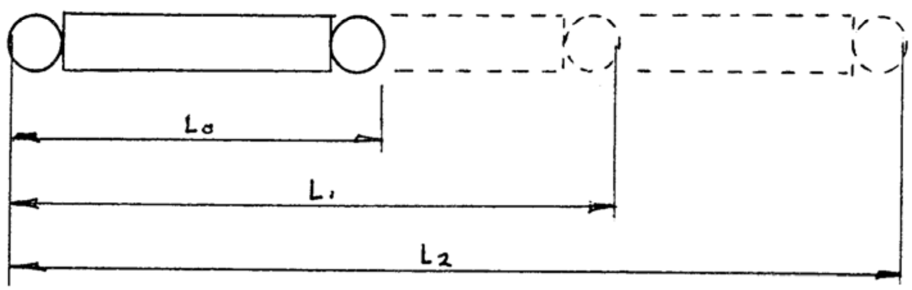
q_b = Bending stress due to P
D_L = Mean coil diameter of loop
d = Wire diameter
c_L = Index of loop = D_L/d
K_L = Curvature correction

$$= \frac{4c_L^2 - c_L - 1}{4c_L(c_L - 1)}$$

Bending stress = bending component + tensile component

$$q_b = \frac{16PD_LK_L}{\pi d^3} + \frac{4P}{\pi d^2}$$

Example 1



1. Design Requirements

Material: Patented carbon steel to BS 1408
Max outside diameter = 0.625 in
Load (P_1) = 5 lbf
Length (L_1) = 3.65 in
Load (P_2) = 17.5 lbf
Length (L_2) = 4.95 in
Initial Tension = 3.95 lbf
English loop

2. Observations

$$\text{Spring rate} = \frac{P_2 - P_1}{L_2 - L_1} = \frac{17.5 - 5}{4.95 - 3.65} = 9.615 \text{ lb/in}$$

The maximum possible load assuming 15% residual range equals P_2 x 115%

17.5 x 1.15 = 29.1 lbf

The required free length

$$L_o = L_2 - \frac{(P_2 - P_1)}{S} = 4.95 - \frac{(17.5 - 3.5)}{9.615} = L_o = 3.494$$

3. Calculations

i) Make certain estimations for the following details:-

Estimate a mean coil diameter (D) = 0.5

Estimate a stress correction factor (K) = 1.2

Estimate a maximum stress (g_m) = 100 000 lbf/in^2

ii) Calculate wire diameter

$$d^3 = \frac{8P_mDK}{\pi q_m} = \frac{8 \times 20.1 \times 0.5 \times 1.2}{\pi \times 100\ 000} = .000307$$

d = .067" or, 0.072" (15 swg) rounding up to the nearest standard wire size.

iii) Check that a maximum stress of 100,000 = 250,000 lb/in^2 is possible with 0.072 dia. wire.

Required tensile = $\frac{100,000}{0.4}$ = 250,000 lb/in^2 or 111 ton/in^2

Using BS 1408 we find that Range 2 has a tensile of 110/120 ton/in^2 for the calculated wire size.

iv) Calculate the active number of coils to give the required spring rate:

$$\text{Active coils, } n = \frac{Gd^4}{8SD^3} = \frac{11.5 \times 10^6 \times 0.072^4}{8 \times 9.615 \times 0.5^3}$$

= 32.1

v) Calculate the free length L_o = (n + 1)d + 2 (D-d)

= (32.1 + 1) 0.072 + 2 (0.5 - 0.072)

= 3.240 in

vi) Compare the free length with that required.
If the calculated free length is larger than required then increase the coil diameter or reduce the wire diameter. This will shorten the spring but increase the stress.

If the calculated free length is shorter than that required reduce the coil diameter or increase the wire diameter.

Alternatively the hooks can be extended.

vii) Go back and change D (and d if required) and recalculate N and L_o until the required free length is achieved.

d 0.072	d .072	d .072
D 0.490	D 0.480	D 0.481
n 34.15	n 36.32	n 36.1
L_o 3.36	L_o 3.5	L_o 3.489

viii) Calculate the new index $c = \frac{0.481}{0.072} = \underline{6.68}$

ix) Calculate the new stress correction factor

$$K = \frac{6.68 + 0.2}{6.68 - 1} = \underline{1.21}$$

x) Calculate the Maximum stress $q_m = \frac{8P_m DK}{\pi d^3}$

$$q_m = \frac{8 \times 20.1 \times 0.481 \times 1.21}{\pi \times 0.072^3} = 79800 \text{ lb/in}^2$$

This relates to a wire tensile of $\frac{79800}{0.4 \times 2240} = \underline{89 \text{ ton/in}^2}$

Thus BS 1408 R1 will be strong enough. Should the stress exceed the allowable wire strength then a stronger material will have to be selected or the spring redesigned incorporating a larger wire and/or a small coil diameter.

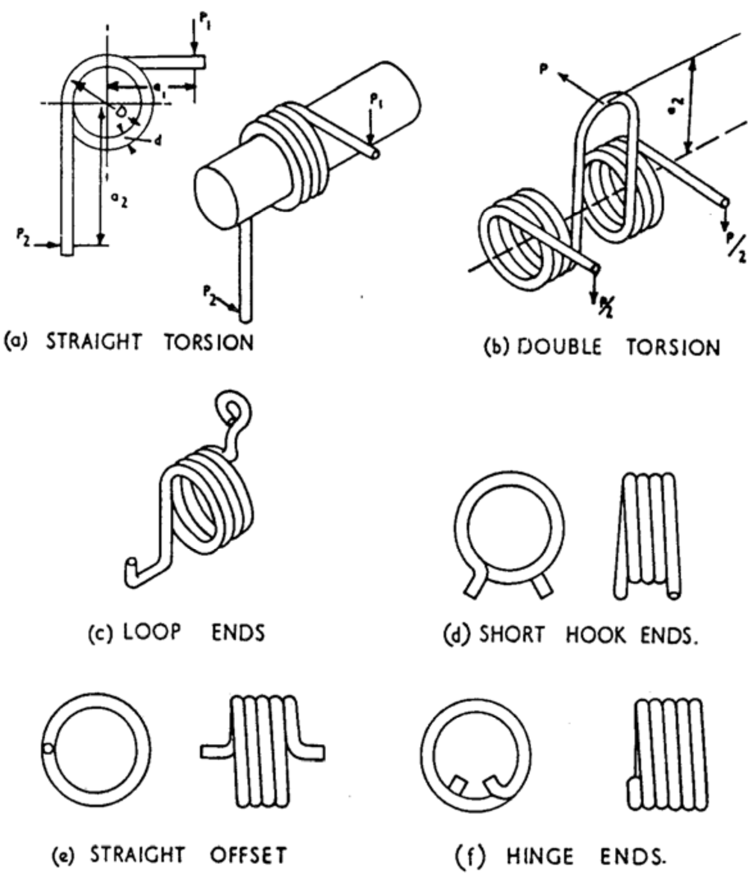
<u>Final Details</u>

d = 0.072"
D = 0.481"
N = 36.1"
L_o = 3.489"

Material:- BS 1408 B Range 1
Stress at 17.5 lb = 69.500 lb/in^2
Stress at 5 lb = 19,850 lb/in^2
Initial Tension Stress = 13900 lb/in^2
Initial Tension as % of wire strength 6.2%.

Part 3 Torsion Springs

Torsion springs differ from both compression and extension springs by the method of operation and calculation. This type of spring may take many forms where the shape of the end is usually determined by the shape of the part to which the spring is to be attached. Some simple examples of end formations are shown below:



(a) STRAIGHT TORSION (b) DOUBLE TORSION

(c) LOOP ENDS (d) SHORT HOOK ENDS.

(e) STRAIGHT OFFSET (f) HINGE ENDS.

For ease of manufacturing one should always aim for the simplest form of leg, ie tangential to the body of the spring.

Torsion springs are loaded by a torque acting about their axis. The direction of operation should always have the effect of winding the spring up, as greater torques can be achieved before overstressing the spring.

Changes in Spring Dimensions

During operation torsion springs will change dimensions, sometimes quite significantly. For example, in a spring operating in the wind up direction the following changes will occur.

i) Number of Coils in the Spring Increases

For one complete turn (360°) of one leg the number of coils in the spring increases by one.

ii) Spring Length Increases

For one complete turn of one leg the spring length increases by one wire diameter.

iii) The Mean Coil Diameter of the Spring Decreases

The percentage reduction in mean coil diameter is the same as the percentage increase in the number of coils. This reduction in diameter can be significant if there are only a few coils. For example:

Free position:

Number of coils in spring = 4
Inside diameter of spring = 25 mm

Wound up position:

One leg is deflected through $^1/_2$ turn (180 deg)

Number of coils in spring = 4.5
Percentage increase in coils = $\frac{4.5 - 4}{4} \times 100 = 12.5\%$

This will also be the percentage decrease in diameter.

Reduction in diameter = $25 \times \frac{12.5}{100} = 3.125$ mm

Therefore inside diameter of spring = 21.875 mm

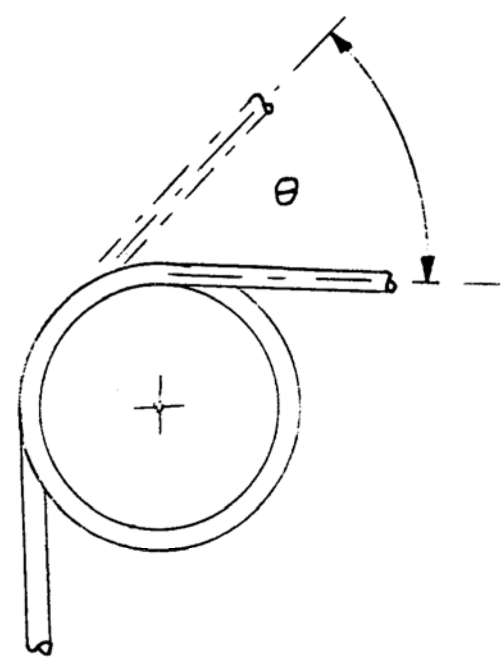
This is a very important fact about torsion springs and must be borne in mind when designing a spring which has to work on a rod of specified diameter. Consequently adequate clearance should always be left between the springs and the mandrel so that the spring does not bind on the mandrel when it is wound up.

Spring Requirements

In general, torsion springs are close-coiled, and any initial tension between the coils should be removed as it can affect the torsional characteristics.

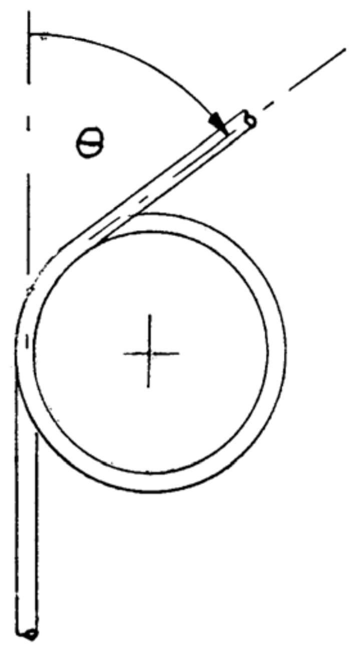
Due to the nature of operation of a torsion spring there are normally two methods of specifying the angular position of the legs after a torque has been applied.

a) Required torque developed after rotation of θ degrees.



This method specifies a torque/angle but does not specify the angular relationship of the legs.

b) Required torque developed at a specified angle.



This method specifies the position of the legs in relationship to one another after a torque has been applied. Hence, if the spring rate is known, the free position of the legs can be determined.

Design Procedure

A logical procedure to obtain the optimum spring design is given in the following section. Let us consider first the notation and formulae;

Notation

L1, L2	= Length of legs (mm, or in)
d	= Wire diameter (mm, or in)
b	= Radial width of rectangular section (mm, or in)
h	= Axial thickness of rectangular section (mm, or in)
D	= Mean coil diameter (mm, or in)
θ	= Angle of rotation (deg)
T	= Applied torque (Nmm, or lbf.in)
N	= Number of coils
Z	= Section modulus = $\frac{\pi d^3}{32}$ for round wire (mm^3, or in^3)
	= $\frac{hb^2}{6}$ for rectangular wire (mm^3, or in^3)
σ	= Bending stress (N/mm^2, or lbf/in^2)
S	= Rotational spring rate (Nmm/deg or lbf in/deg)
E	= Youngs modulus

Values of Youngs modulus for a range of spring materials are given in the table below:

Modulus Values for Spring Material

	E (kN/mm^2)	E (lbf/in^2)
Hard drawn carbon steel	200	30×10^6
Carbon steel for hardening and tempering	200	30×10^6
Silicon manganese steel	200	30×10^6
Chromium vanadium steel	200	30×10^6
Martensitic stainless steel	200	30×10^6
Austenitic stainless steel	180-190	$26.5\text{-}28 \times 10^6$
Phosphor bronze	100	15×10^6
Hard drawn brass wire	100	15×10^6
Copper beryllium	130	18.5×10^6
Monel	180	26×10^6
Inconel X750	210	31×10^6
Nimonic 90	235	34×10^6
Titanium alloys	110-130	$16\text{-}19 \times 10^6$

Design Stress

The bending stress (σ) is related to the applied torque (T) by the following:-

$$\sigma = \frac{T}{Z}$$

Hence, for round wire material:-

$$\sigma = \frac{32 \times T}{\pi d^3}$$

It should be noted from the above formula that, unlike compression and extension springs, there is no mean coil diameter term. This means that stress is independent of mean coil diameter and thereby simplifies the design procedure for torsion springs.

Deflection Under Torque

The angular rotation (deflection) under a given applied torque is given by:-

$$\theta = \frac{11520 \times T}{E \times \pi^2 \times d^4} \left[\left(\frac{L1 \times L2}{3} \right) + (N \times \pi \times D) \right]$$

By re-arranging the above formula it is possible to give a relationship for the rotational spring rate as follows:-

$$S = \frac{E \times d^4 \times \pi^2}{11520 \times \left[\left(\frac{L1 + L2}{3} \right) + (N \times \pi \times D) \right]}$$

Maximum Allowable Stresses

Compression springs are stressed in torsion but torsion springs are stressed in bending. Consequently, operating stresses for torsion springs can be much higher than those for compression springs. The maximum allowable stress that a spring can withstand before taking a permanent set is dependent upon the properties of the material. For example, a torsion spring manufactured from a patented wire to BS 1408 can achieve stresses in the order of 70% of the tensile strength of the material without any heat treatment (60% for EN58A and 70% for oil hardened and tempered wire to BS 2803). With conventional low temperature heat treatment the stress levels can be raised quite significantly to 110% for BS 1408 (100% for EN58A) and 140% for oil hardened and tempered wire when stress relieved at 250°C.

Table for maximum allowable stress in torsion springs (σ_{max}) as a percentage of the UTS.

	'As Coiled'	After LTHT
Cold drawn carbon steel	70% UTS	110% UTS
Cold drawn stainless steel	60% UTS	100% UTS
Oil hardened and tempered steel	70% UTS	140% UTS

Although higher stresses can be achieved by heat treatment, this operation will affect the position of the legs. The springs will either wind up (BS 1408, BS 2803) or unwind (EN58A) dependent upon the material from which they were manufactured.

Unlike compression springs, torsion springs have no physical stop which prevents overstressing and so they should always be designed with a residual range of 15%.

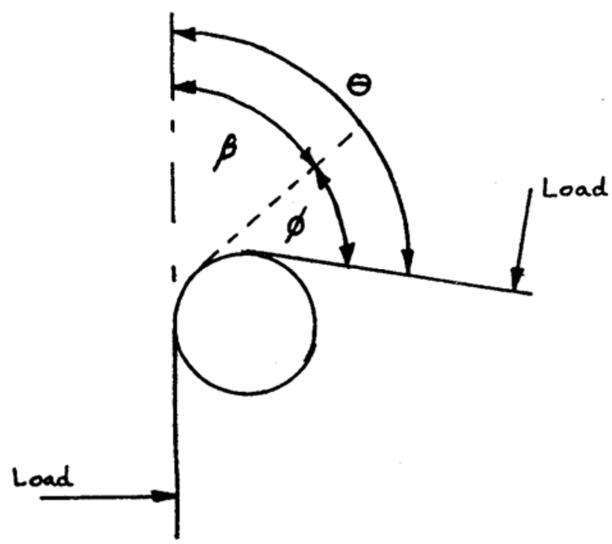
ie Maximum design stress = maximum allowable stress x 0.85. Hence the maximum operating torque (T max) is given by:-

$$T_{max} = \frac{d^3 . \sigma_{max} \times 0.85}{32}$$

Design Example

Torsion Spring Requirements

Maximum Outside Diameter	= 11 mm
Minimum Inside Diameter	= 7.5 mm
Maximum Torque	= 126 Nmm at a leg angle of 270°
Maximum Length	= 11 mm
Length Leg 1	= 7 mm
Length Leg 2	= 8 mm
Torsional Rate	= 0.7 Nmm/Deg



1. Observations

Deflection from free at max torque:

$$\phi = \frac{\text{Torque}}{\text{Rate}}$$

$$= \frac{126}{0.7} = 180 \text{ deg}$$

Angular relationship of legs in free position

$\beta = \theta - \phi = 270 - 180 = 90$ deg $= {}^{1}/_{4}$ turn

Total coils in free position = Integer + ${}^{1}/_{4}$

2. Determine Wire Diameter

Estimate a wire size, say 1.0 mm

$$\text{Maximum allowable stress} = \frac{32 \times T}{\pi \times d^3 \times 0.85}$$

$$= \frac{32 \times 126}{\pi \times 1^3 \times 0.85}$$

$$= 1510 \text{ N/mm}^2$$

Consequently we need a material with a tensile strength of

$$\frac{1510}{0.7} \text{ N/mm}^2$$

BS 5216 Grade 4 will be suitable. If the stress was too high then we would have to revise our estimate to a larger wire size.

3. Estimate Number of Coils in Free Position

We have already determined that total coils in free position = Integer + $^1/_4$.

So let us assume total coils of $8^1/_4$.

This gives a spring length of

$(8.25 + 1) \times d = 9.25$ mm in free position

$(8.25 + 0.5 + 1) \times d = 9.75$ mm in wound position

The latter value is the critical one as it must be less than the specified value of 11 mm.

In this example the value is OK: if this had not been the case then the total number of coils would have needed reducing.

4. Calculate Spring Diameter

$$\text{Rate } (R) = \frac{E \times d^4 \times \pi^2}{11520 \times \left[\frac{L1 + L2}{3} + N \times \pi \times D\right]}$$

$$D = \frac{E \times \pi \times d^4}{11520 \times N \times R} - \frac{L1 + L2}{3\,N\pi}$$

$$= \frac{2 \times 10^5 \times \pi \times 1^4}{11520 \times 8.25 \times 0.7} - \frac{7+8}{3 \times 8.25 \times \pi}$$

$$= 9.44 - 0.19$$

$$= 9.25 \text{ mm}$$

Outside diameter free position = 9.25 + 1.00 =10.25mm
Inside diameter free position = 9.25 - 1.00 = 8.25mm
Inside diameter wound position = $\left[1 - \frac{0.5}{8.25}\right] \times 9.255 - 1 = 7.69$mm

These details fit within the specifications. However, if the spring diameter had been too small, one or both of the following options could have been selected:

i) Increase wire diameter and repeat from section 2.
ii) Reduce number of coils and repeat from section 4.

If, on the other hand, the spring diameter had been too large, the following options would have been possible:

i) Reduce wire diameter and repeat from section 2.
ii) Increase number of coils and repeat from section 3.

5. Final Details

Material	BS 5216 M4
Wire Diameter	1.0 mm
Mean Coil Diameter	9.25 mm
Total Coils	8.25
Stress at 126 Nmm torque	1283 N/mm^2

Part 4 Nested Springs

A Introduction

Nested springs are normally employed when a single spring cannot be designed to satisfy the load at length requirements due to restrictive space limitations. Nested springs make better use of the space available than a single spring, thus yielding a higher volumetric efficiency which can result in one or more of the following advantages:

i) The stress is reduced
ii) The outside diameter is decreased
iii) The length is reduced

The effect of using section material gives the same advantages as those described above except the improvement in volumetric efficiency will be less marked.

B Design Considerations

1) The overall requirements must be split to each of the springs in the nest. For a nest of two springs the total applied load should be split in the ratio 2:1.

   For example;

   Required load 2100 N.

   Load to be supported by the outer spring $\frac{2}{2+1} \times 2100 = 1400$ N

   Load to be supported by the inner spring $\frac{1}{2+1} \times 2100 = 700$ N

   For a nest of three springs the load should be split in the ratio 4:2:1.

   For example;

   Required load 2100 N

   Load to be supported by outer spring $= \frac{4}{4+2+1} \times 2100 = 1200$ N

   Load to be supported by middle spring $= \frac{2}{4+2+1} \times 2100 = 600$ N

Load to be supported by inner spring $= \frac{1}{4 + 2 + 1} \times 2100 = 300N$

2) The solid length of the outer spring should be made greater than the solid length of the inner spring, by approximately $^1/2$ a wire diameter.

3) Adjacent springs in the nest should be wound to the opposite hand so that they do not entangle.

4) Sufficient clearance should be allowed between adjacent springs to minimise any possible interference. The outside diameter of the adjacent inner spring can be calculated from the inner diameter of the outer spring using the following formula:

$(D_o)_i = (D_i)_o \times x$

$(D_o)_i$ = Outside diameter of inner spring

$(D_i)_o$ = Inside diameter of outer spring

x = Clearance factor taken from table below

Inside Diameter of Outer Spring (in)	Clearance Factor x
.81	0.92
1.08	0.93
2.13	0.94
3.16	0.95
4.18	0.96
5.21	0.96
6.23	0.96
7.25	0.97

5) The inner spring should be checked for buckling - if this is stable, then the outer spring will also be stable.

C Design Procedure

Using the required load characteristics determined in step 1) for the outer spring, follow the standard procedure for the design of compression springs

Having designed the outer spring, use the table in step 4) to calculate the outside diameter of the inner spring. Using the load requirement calculated in step 1) for the inner spring, again follow the standard design procedure. Compare the solid length of this spring with the outer and make any adjustments to wire diameter and spring diameter as are necessary to obtain the correct solid length. After any change to spring diameter check clearance between springs. Once an acceptable design has been established for the outer spring pair, the procedure is repeated for any further springs in the nest.

Part 5 Rectangular Section Compression Springs

Rectangular section springs are normally employed when a single spring cannot be designed to satisfy the load at length requirements.

Design Considerations

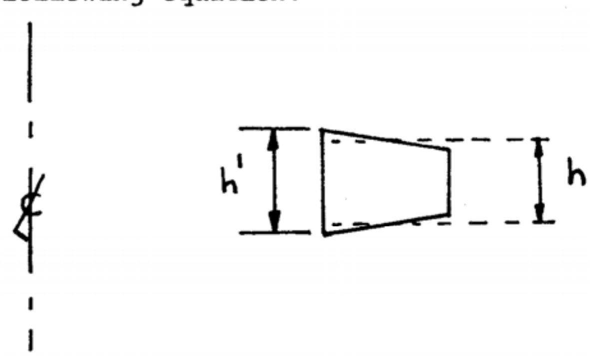
In general, the use of section material can enable springs of a lower stress to be designed. This is because more material can be fitted into the available space.

Under certain conditions, notably for large relative deflections, a change from round to rectangular section material will enable a spring design to become stable as regards buckling.

Shaped wire is not as good in fatigue as round wire because in general shaped wire has an inferior surface finish, and the reduction in stress which can be achieved does not compensate for the reduced fatigue properties.

Rectangular or square section material does not come in standard wire sizes. It is usually made to order and is therefore difficult to obtain, especially in small quantities.

When springs are coiled, the section of the wire changes shape. This does not matter when the wire is round, but can significantly affect the solid length when the wire is rectangular. The amount of 'upsetting' is easy to estimate by the following equation.



Where $h^1 = h\left(1 + \frac{k}{c}\right)$

h^1 = height after coiling

h = original thickness

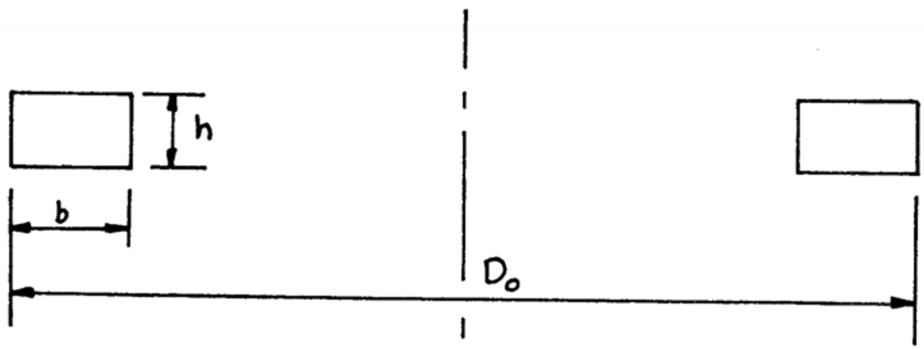
c = Spring Index = D/b

k = 0.3 for cold drawn or prehardened and tempered materials
0.4 for annealed or hot coiled materials

It is often necessary under these circumstances to use 'Bevel section' or 'keystone' material, which has been shaped in the opposite direction to allow for the upsetting.

Design Equations

The design equations are similar but more complicated than those for round section material, including factors which vary according to the shape of the spring.



$$\text{Rate} = S = \frac{\mu b^2 h^2 G}{nD^3}$$

$$\text{Stress} = \tau = \frac{\lambda P}{bh}$$

Where b = Radial width of section

h = Axial height of section

m = $^b/_h$ = shape ratio

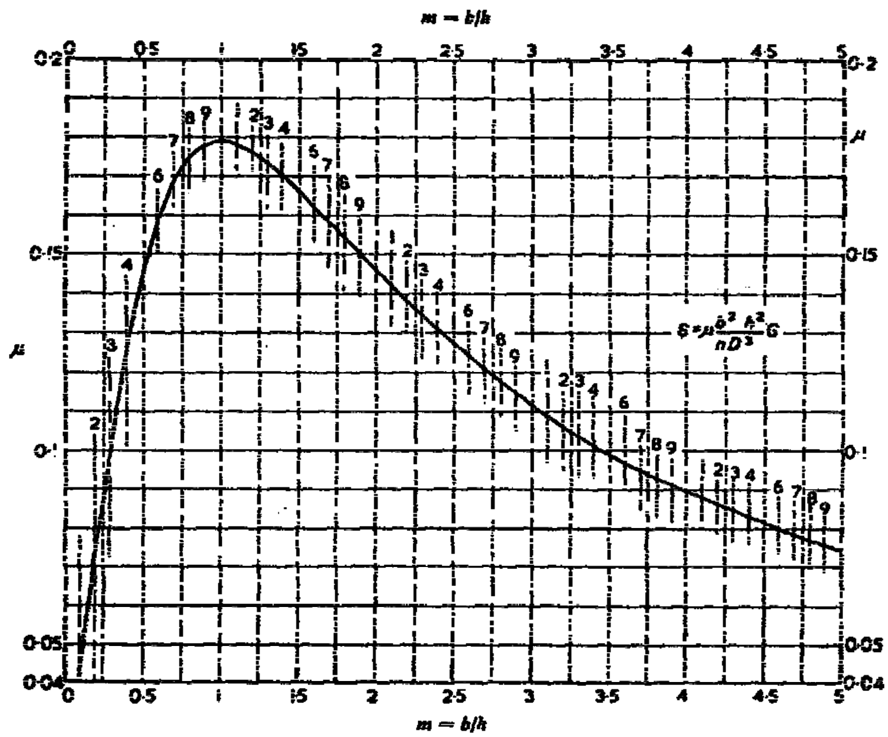
P = Axial load

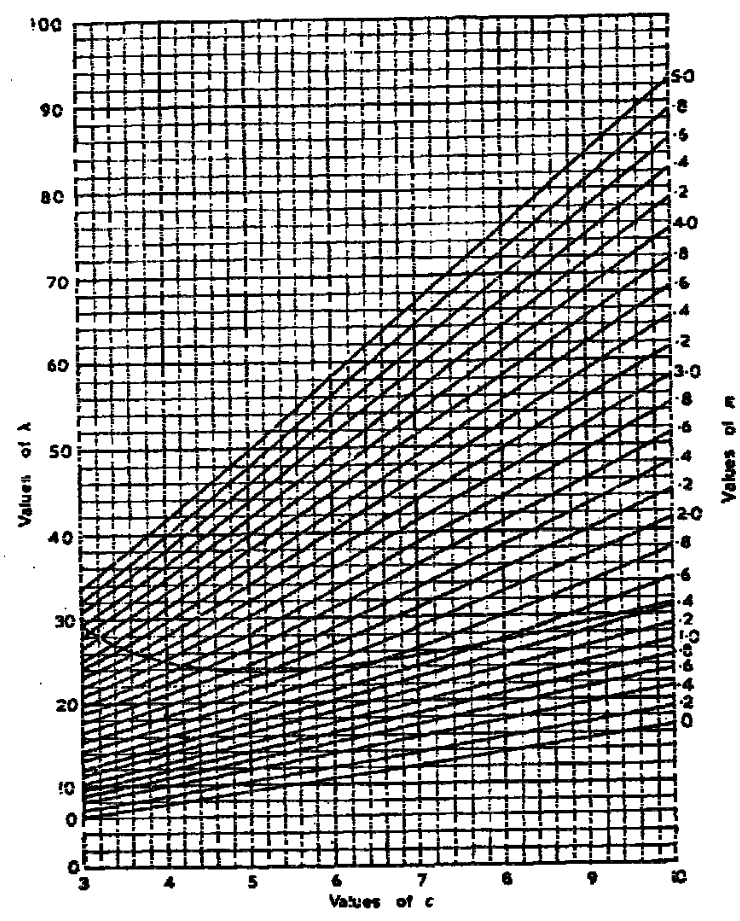
D = D_o - b = Mean coil diameter

n = Number of active coils

G = Modulus of rigidty

μ and λ are functions of c and m which can be obtained from the following graphs.


m = b/h
μ
0·2
0·15
0·1
0·05
0·04
0
0·5
1
1·5
2
2·5
3
3·5
4
4·5
5


Values of λ
Values of n
Values of c
100
90
80
70
60
50
40
30
20
10
0
3
4
5
6
7
8
9
10

Design Procedure

The design procedure is similar to that for round wire compression springs as described in Part 1 , except for the following points.

i) Section size assumed;
ii) Stress and rate formula replaced by those above;
iii) In order to determine allowable stresses the section size needs to be related to a standard wire size, this is done by the following formula:

$$\text{equivalent wire diameter} = \sqrt{\frac{4(h \times b)}{\pi}}$$

iv) Section ratio can be adjusted to alter solid length of spring.

Design of Conical Springs

(a) Introduction

Conical springs are infrequently used due mainly to the difficulties of design and problems of manufacture. However there are two characteristics of conical springs that can be used to great advantage. These characteristics can be incorporated in a design individually or in combination and are as follows:-

i) Non linear rate.

Since the coil diameter varies in a conical spring the flexibility of each coil is different; the coils with larger diameters being more flexible than the smaller diameter coils. Consequently the larger coils deflect further under the action of a compressive force, and if the vertical pitch is constant, the larger coils will make contact with each other before the smaller coils.

Up to the point at which the largest coil first makes contact, the spring rate is linear. From this transition point, however, the rate increases as more large diameter coils become inactive by making contact with the adjacent coil.

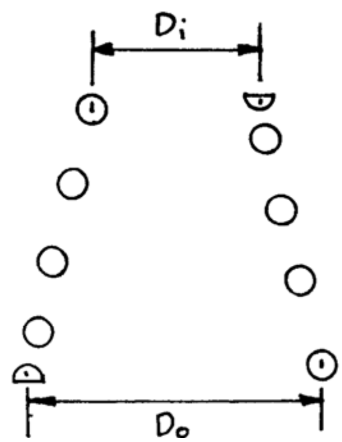
ii) Reduced solid lengths.

By designing a spring with one end larger than the other it is possible to make coils seat slightly inside each other, thus reducing the solid height of the spring. In fact, if the diametral space is available, it is possible to design a spring with the solid height equal to the wire diameter.

Due to these factors accurate calculation for a spring's characteristics are involved and extremely tedious by hand. Hence many methods of approximation aimed at simplifying the calculations are available, these varying in degrees of inaccuracy. It is not proposed here to list these methods but to show the procedure using accurate equations. The widespread use of computers today enables simple use of these equations.

(b) Design Considerations

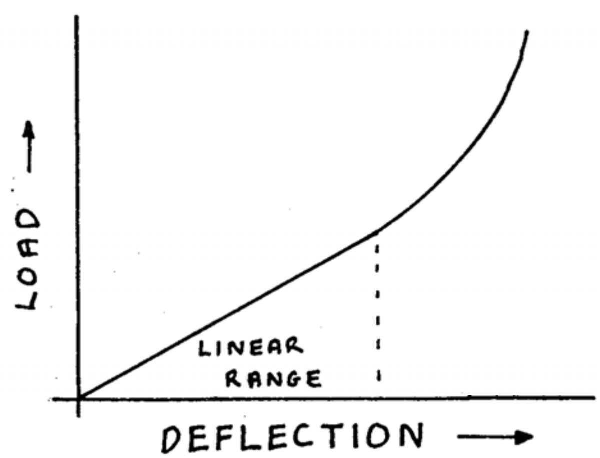
Nomenclature



D = Mean coil diameter of any coil
Do = Mean coil diameter of largest coil
Di = " " " " smallest coil
N = Number of active coils
P = Applied load
d = Wire diameter
c = Spring index (D/d)
k = Correction factor $\frac{c+0.2}{c-1}$
G = Modulus of Rigidity
S = Spring rate
r = radius of element
L = free height of active coils

Spring Rate

The rate of a conical spring can easily be calculated for the linear portion of the load deflection characteristic.



The formula for linear rate is

$$S = \frac{Gd^4}{n(Do + Di)^3}$$

Stress

The stress in the spring varies through the spring and is calculated as follows:-

$$q = \frac{8PD}{\pi d^3} K$$

This is the general formula and D is the mean coil diameter of the coil in which the stress is being determined. Consequently the correction factor must be based upon the index of that coil.

The maximum stress in the spring will always occur in the largest active coil. Hence, during the linear portion of the rate line, maximum stress will occur in coil Do. However, as the spring is compressed and coils start contacting, the maximum stress will occur in the largest remaining active coil. The last remaining active coil before the spring goes totally solid is the smallest diameter coil. Hence the solid stress of a spring is:

$$qs = \frac{8PsDi\ K}{\pi d^3}$$

$$\text{where } K = \frac{Di/d + 0.2}{Di/d - 1}$$

If this stress exceeds the elastic limit of the material then part of the spring will be prestressed. The amount of prestress will vary through the spring. The geometry of the spring will be significantly affected by this feature. If the spring is coiled with constant vertical pitch then, after prestressing, the pitch will be reduced by varying amounts through the spring. If a constant vertical pitch is required after prestressing, the spring must be coiled with an increasing pitch towards the small diameter end.

Deflection

This is calculated by considering each coil in the spring to be divided into a number of segments (e). The more segments the more accurate the result but the more tedious the calculation.

$$\text{Deflection of one element in a spring} = \frac{64PDr^3}{Gd^4e}$$

The relationship for calculating the radius of the nth element is as follows:

$$r = \frac{Di}{2} + \frac{n\ (Do-Di)}{2N \times e}$$

Thus,

$$\delta_e = \frac{64P}{Gd^4e}\left[\frac{Di}{2} + \frac{n\ (Do-Di)}{2Ne}\right]^3$$

However, this equation does not take account of elements making coil-to-coil contact. Consequently a limiting factor has to be placed upon the calculated deflection to prevent it exceeding that which is physically possible for the spring design.

The maximum possible deflection for each element can be calculated from the following equation.

$$\delta_{max} = \frac{L-d-\sqrt{(N-1)^2 d^2 - \left(\frac{Do-Di}{2}\right)^2}}{Ne}$$

Or, if the spring has been designed to lie completely flat at solid:

$$\delta_{max} = \frac{L-d}{Ne}$$

If the calculated deflection δ_e exceeds the maximum possible for that element δ_{max} then the value δ_{max} is taken. The total deflection of the spring under an applied load is obtained by summing all the individual deflections for each element.

(c) Design Procedure

i) Estimate large and small diameters based upon dimensions of required end fitments

ii) Use minimum spring diameter and estimated solid load to determine solid stress and estimate of wire diameter. Compare stresses with those allowable in the same way as for parallel sided compression springs.

iii) With the required rate for the linear portion calculate the number of coils.

iv) Produce load deflection characteristics of spring by using specified load and calculate deflection of each element in turn. When the calculated deflection of an element is greater than the maximum allowed, that element has gone solid and so the maximum allowed deflection only should be considered. Repeat this for all specified loads.

v) Maximum stress in spring at any time will always occur at largest active segment. This will always be the one giving the largest deflection without exceeding the limiting factor.

vi) If load deflection characteristics do not match the required then then re estimate dimensions of spring and start again at i)

vii) Check solid stress of spring against that allowable.

Conical Springs Characteristics

As stated, conical springs do exhibit a non linear portion of the load deflection curve when, upon deflection, the coils close and hence the number of active coils decreases. The point at which this occurs is called the transition point.

However, another practical application for conical springs, which does not require a non linear load length characteristic, is the simple expedient of reducing the solid length of the spring. This is achieved if the spring is designed so that the coils fit in one another without touching as the spring is deflected. This ensures that the working range of the spring is within the linear portion of the load deflection curve; and the design formulae become very easy to use since the design is exactly the same as for a parallel sided spring, except that the mean coil diameter used in rate calculations is an average of the mean coil diameters at either end of the spring, and the mean coil diameter used in stress calculations is the largest mean coil diameter.

Ensuring that the transition deflection is greater than the deflection to the minimum working height is best achieved during manufacture when setting the coiling machine. Visual inspection of coiled samples under load test will reveal whether or not all the coils are still active (i.e. the spring is in the linear portion) at the minimum working height. If this is not the case, small alterations can be made to the pitch tool control at this stage, in order to extend the linear range.

A further point which requires clarification is that the formulae presented in the first part for the limiting value of the maximum deflection of each element hold true only for springs with a constant vertical pitch. This is the general form of an unprestressed spring coiled on a single point coiling machine with a push-type pitch tool. To obtain a constant vertical pitch on a two point coiling machine with a wedge type pitch tool is extremely difficult. The general form produced by such a machine is of increasing pitch at increased coil diameters. This effect tends to increase the linear portion of the load/deflection curve which is an advantage if a linear rate is required. If a specific non-linear rate is required, it should be borne in mind that this can be achieved using a parallel sided compression spring with variable pitch as an alternative to a conical spring.

Conical Spring Design Example

To design a constant vertical pitch conical spring to suit the following requirements.

i) Both ends to fit over a 19mm dia. rod
ii) One end to fit inside a 25mm dia. recess
iii) One end to fit inside a 65mm dia. recess
iv) Solid height to be less than 5mm
v) The rate must be linear between the following load/lengths:-

(a) At a spring height of 47.5mm load to be .92 N
(b) At a spring height of 36.0mm load to be 5.18 N

vi) Load at 15mm height must be less than 22 N

1. Calculate the linear rate and free length as follows:-

(a) Linear rate $= \frac{5.18 - .92}{47.5 - 36} = .37$ N/mm

(b) Free length of spring $= 47.5 + \frac{.92}{.37} = 50$ mm

2. Assume the following:-

i) Wire diameter = 2.24mm
ii) Mean coil diameter large end = 59.75mm (Do)
iii) Mean coil diameter small end = 21.75mm (Di)

NOTE:- In the design of conical springs the determination of the wire diameter is an iterative process which involves a choice of the diameter (usually based on experience) and working through the design procedure in order to determine the solid stress. If this stress is too high, a new estimate of the wire size is necessary and the procedure repeated. For the sake of brevity, the size chosen in this example is known to be satisfactory.

3. Using the wire size, calculate the required number of coils to deliver the specified spring rate in the linear rate zone as follows:-

$$S = \frac{Gd^4}{N\,(D_o + D_i)^3}$$

Therefore

$$N = \frac{Gd^4}{s(D_o + D_i)^3} = \frac{79300 \times 2.24^4}{.37 \times (59.75 + 21.75)^3}$$

Therefore N = 10 active coils

Therefore Total coils (N_t) = 12

4. The limiting factor on deflection of each element in a constant pitch spring must now be calculated.

First determine if the coils of the spring will seat inside each other thus producing a solid height of the spring equal to the wire diameter.

$$\text{space available} = \frac{\text{Outside Dia. large end} - \text{Inside Dia. small end}}{}$$

$$= 61.99 - 19.51$$
$$= 42.48 \text{ mm}$$

This available space would only accommodate 9 coils of 2.24 mm diameter wire so the spring will not lie completely flat. For ease of design and manufacture it would be desirable for the spring to lie flat when fully compressed. This may be achieved by re-selection of mean coil diameters. However, in this particular case, the restriction on the available space (ie recess diameter of large end and the rod diameter for the small end of the spring) does not allow this option. Alternatively, the selection of a smaller wire size with the same spring diameters may produce a design that will lie flat.

In order to demonstrate the formula for both conditions of conical spring the example first deals with the situation where the spring will not lie flat (assumptions as under para 2) and later the situation where the spring will lie flat.

The limiting factor for each element of the conical spring which does not lie flat is calculated as follows:-

$$\delta_{max} = \frac{L - d - \sqrt{(N-1)^2 d^2 - \left(\frac{D_o - D_i}{2}\right)^2}}{Ne}$$

$L = L_o - d$, for a closed and ground spring
Assume e = 4 (i.e. 4 elements per coil)

$$\delta_{max} = \frac{47.76 - 2.24 - \sqrt{9^2 \times 2.24^2 - \left(\frac{59.75 - 21.75}{2}\right)^2}}{10 \times 4}$$

$= 0.97\text{mm}$

5. Next, the solid stress in the spring is calculated from the solid load.

The solid load in the spring will be that required to close the last active element. The last active element will be the element with the smallest coil diameter since this will be the least flexible element (i.e. the 1st element in the summation). The deflection of this element (   ) is therefore set equal to the limiting value calculated in step 4 (i.e. 0.97 mm)

$$(\delta 1)_{max} = .97 = \frac{64P}{Gd^4e}\left[\frac{D_i}{2} + \frac{n\ (D_o - D_i)}{2\ Ne}\right]^3$$

For the first element n = 1. Hence, by rearranging the above formula,

$$P_s = \frac{.97 \times 79300 \times 2.24^4 \times 4}{64 \times \left[\frac{21.75}{2} + \frac{1 \times (59.75 - 21.75)}{2 \times 10 \times 4}\right]^3} = 83N$$

The spring index of the last active element (i.e. first element in summation) is given by:-

$$C_1 = \frac{21.75}{2.24} = 9.7$$

The solid stress of the spring can now be determined since this is the stress in the last active element calculated as follows:-

$$\tau_s = \frac{8P_sD_iK}{\pi d^3}$$

$$\text{Therefore } \tau_s = \frac{8 \times 83 \times 21.75}{\pi \times 2.24^3}\left(\frac{9.7 + .2}{9.7 - 1}\right)$$

$$\text{Therefore } \tau_s = 465N/mm^2$$

For unprestressed springs BS 5216 Grade 1 will be acceptable.

If the stress had been too high then it would have been necessary to perform one or both of the following:-

i) Select a larger wire size and return to step 2.
ii) Increase the linear range of the spring by increasing the diameter of the small end or by reducing the diameter of the large end.

6. A check must now be made to ensure that the second load length at 36.0mm is within the linear range of the load/deflection curve. The position at which the load length characteristic becomes non-linear is known as the transition point and occurs when any one of the elements first makes coil-to-coil contact with a neighbouring element. In order to determine the transition point, set the deflection of the largest coil diameter element equal to its maximum value and calculate the load, since the largest coil diameter element is the most flexible and will therefore make contact first.

Therefore

$$\delta_{40} = 0.97 = \frac{64P}{Gd^4e}\left[\frac{D_i}{2} + \frac{n\ (D_o - D_i)}{2Ne}\right]^3$$

For the largest diameter element (the 40th element) $n = 40$, the transition load (P_t) can be calculated by re-arranging the above formula as follows:-

$$\text{Therefore } P_t = \frac{0.97 \times 79300 \times 2.24^4 \times 4}{64\left[\frac{21.75}{2} + \frac{40 \times (59.75 - 21.75)}{2 \times 10 \times 4}\right]^3}$$

$$P_t = 4.54N$$

By comparison with the specified load at 36.0mm, it can be seen that the specified load is less than the transition load (P_t) so the design is acceptable on this point. However, if the transition load had been less than the required load, one or more of the following would have been necessary:-

i) Select a smaller wire diameter and return to step 2.
ii) Increase mean coil diameter of small end and return to step 2.
iii) Increase mean coil diameter of large end and return to step 2.

7. The next step is to calculate the solid height of the spring. For a conical spring with closed and ground ends that does not lie flat this is given by:-

$$L_c = \sqrt{(N_t \times d)^2 - \left(\frac{D_o - D_i}{2}\right)^2}$$

$$L_c = \sqrt{(12 \times 2.24)^2 - \left(\frac{59.75 - 21.75}{2}\right)^2}$$

$$L_c = 19.0mm$$

This is greater than the specified value so one or more of the following <u>must</u> be performed:-

i) Reduce the wire size and return to step 2
ii) Increase the outside diameter of the large end coil and return to step 2.

8. The wire diameter is reduced to 2.12mm and the mean coil diameter of the larger end is increased to 61.5mm in order to reduce the solid length. Hence the new design is as follows:-

Wire diameter 2.12mm
Mean coil diameter large end 61.5mm
Mean coil diameter small end 21.75mm

9. Based upon these new values, the following details are calculated as before from steps 3 to 7.

i) Active coils = 7.5
ii) Total coils = 9.5
iii) In the diametral space available it is possible to fit 9.5 coils, so this spring will lie flat. The limiting deflection is given by the following formula:-

$$\delta_{max} = \frac{L - d}{Ne} = \frac{47.88 - 2.12}{7.5 \times 4} = 1.525 \text{ mm}$$

iv) solid load = 99.4N
solid length = 2.12 (wire diameter) O.K.
v) solid stress = 656 N/mm^2

Hence BS 5216 Grade 1 will be acceptable for an unprestressed spring.
vi) Transition load = 5.25N.
This is acceptable since it exceeds the second load required.

vii) Solid length = 2.12mm (this is the wire diameter for a spring lying flat).

10. The final requirement of the specification, that the load at 15mm working height is less than 22N, is checked by calculating the spring height under a 22N load.

This is obtained by summing the deflections of each element under a 22N load. The deflection for each element is given by the formula,

$$\delta_e = \frac{64P}{Gd^4e} \cdot \left[\frac{D_i}{2} + \frac{n\,(D_o - D_i)}{2Ne}\right]^3$$

For each element the calculated deflection from the above formula is compared with the limiting factor. If the calculated value exceeds the limiting value, the latter is used in the summation for total deflection. For elements 16-20 the results of these calculations are shown below.

Element No	Calculated Deflection (mm)	Deflection used in summation (mm)
16	1.981	1.525
17	1.798	1.515
18	1.626	1.525
19	1.466	1.466
20	1.317	1.317

In this example the calculated deflections per elements 1-18 exceed the limiting factor of 1.525mm, so the total deflection under 22N load can be expressed as:-

$$\text{deflection @ 22N} = (18 \times 1.525) + \delta_{19} + \delta_{20} + \delta_{21} + \ldots\ldots + \delta_{30}$$
$$= 37.32$$

Hence the spring height under load 22N = 50 - 37.32
= 12.68mm

So the load at 15mm height will be less than 22N.

Variable Pitch Conical Springs

The example covered the design of a constant vertical pitch spring. The design procedure for variable pitch springs is similar (providing the pitch varies linearly), except for the following points:-

i) The limiting deflection for each element will now be different. The value is calculated as in a constant pitch spring and then multiplied by the following factor:-

$$\frac{n \times \left(\frac{P_o - P_i}{Ne}\right) + hi}{\left(\frac{P_o + P_i}{2}\right)}$$

where n is the element being considered

Po is the pitch of the largest diameter coil.
Pi is the pitch of the smallest diameter coil.
N is the number of active coils
e is the number of elements/coil

ii) Due to the variable pitch in the spring the largest diameter coil may no longer be the first to make coil-to-coil contact. Consequently the first and last coils to close must be determined by calculation.

For each element, the load at which the element deflection equals its associated limiting deflection is calculated. The element with the lowest load is then the first to close and the transition point has been determined (the load being the transition load). The element with the largest load to closure is the last to close and the coil diameter of this final element is used in the calculation for solid stress.

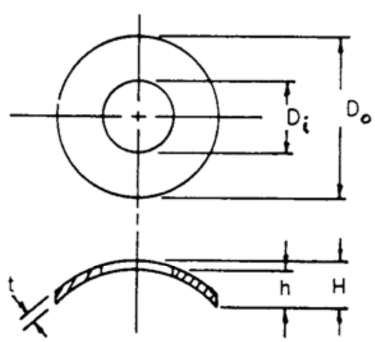
Spring Washers

There are many designs and shapes of spring washers in use but the most common are curved washers and wave washers. These washers are usually used in thrust loading applications where small deflections are required and where radial space is limited. Due to the simplified nature of the formulae used for predicting loads in washers and to the presence of significant frictional forces, the load tolerances should never be less than +/- 20%.

Curved Washers

These are well suited to light thrust loads, giving a spring rate which is near to linear. Allowance should be made for the expansion of the diameter as the washer becomes compressed, and generally the deflection should be limited to 80% of the height of the washer.

The simplified formulae given below are only approximations but are sufficiently accurate for most design purposes:



The formulae for calculating the load (P) and the bending stress (f) are:-

$$\text{Load } (P) = \frac{4E\delta t^3 (D_o - D_i)}{D_o^3}$$

$$\text{Stress } (f) = \frac{1.5PD_o}{t^2(D_o - D_i)}$$

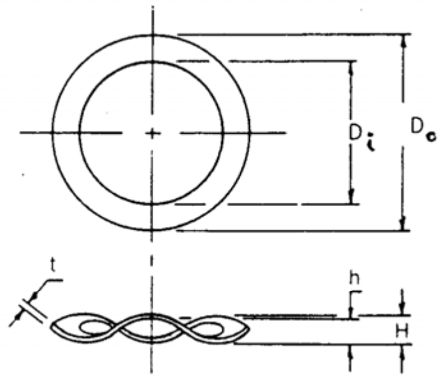
where E = Modulus of elasticity

δ = Deflection

t = Material thickness

Wave Washers

These are suitable for light to medium loads and again the spring rate in the range 20% to 80% of the available deflection is close to being linear. Like curved washers, the load tolerance cannot be held tighter than +/- 20% and, on loading, the washer increases in diameter although less so than with curved washers. The form of a wave washer is shown in the sketch below for a washer having three full waves.



To evaluate the load (P) and bending stress (f) the following equations should be used:-

$$\text{Load (P)} = \frac{E\delta bt^3N^4}{2.4D^3} \cdot \left(\frac{D_o}{D_i}\right)$$

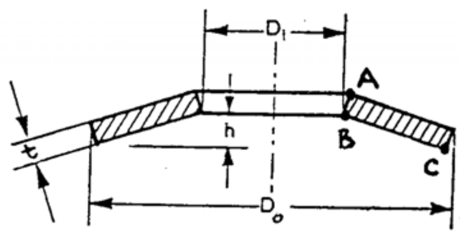
$$\text{Stress (f)} = \frac{3\pi PD}{4bt^2N^2}$$

Where E = Modulus of elasticity
δ = Deflection
t = Material thickness
N = Number of complete waves
D = Mean diameter = $\frac{D_o + D_i}{2}$

b = Radial width of material = $\frac{D_o - D_i}{2}$

Disc Springs

Disc springs or, as they are sometimes called, "Belleville Washers" consist of a coned disc with a central hole, as illustrated below. More often than not the material thickness is constant across the annular section.



Disc springs are particularly useful where high loads need to be accommodated with little deflection of the spring element. Other features and advantages of disc springs include a wide range of non-linear load-deflection characteristics depending on the choice of cone height to material thickness, diameter, flexibility in changing the load-deflection behaviour by stacking the discs in series or in parallel and self damping due to friction, particularly when arranged in parallel.

From a design aspect, one of the most important considerations is the load-deflection behaviour, which is dependent on the dimensions of the inside and outside diameters, material thickness and the free cone height of the disc spring. In general the ratio of the outside to inside diameter varies from about 1.5 to 3.5 and this will have an effect on the load-deflection characteristics. The most influential dimensions, however, are the cone height and disc thickness which can be altered to provide a variety of different shaped load-deflection curves. This is shown in the diagram below for single disc springs having cone height to thickness ratios of 0.5, 1.5 and 2.75, supported in such a manner as to allow deflections greater than cone height to be achieved. With h/t ratios around 1.5, a considerable portion of the load-deflection curve is near-horizontal and this feature can be exploited in certain applications where near 'zero rate' is required.

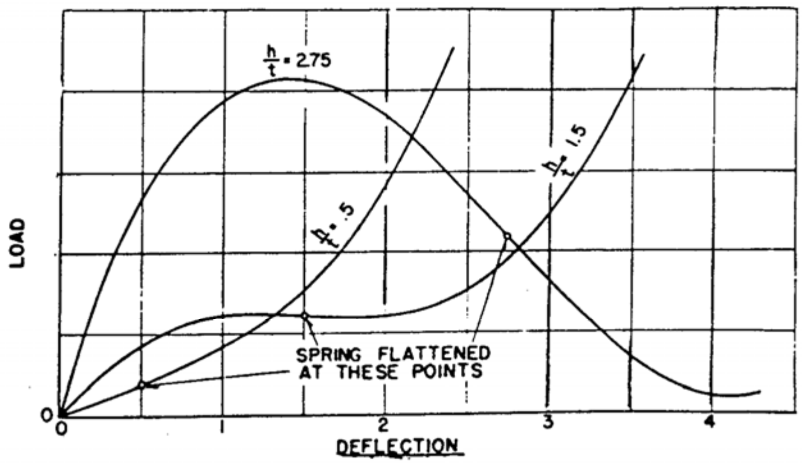


Fig 1 Load-Deflection Curves for Various h/t Ratios

By the use of disc springs stacked in series, in parallel, or in combination with one another, further variations in the load-deflection characteristics can be achieved (Fig 2).

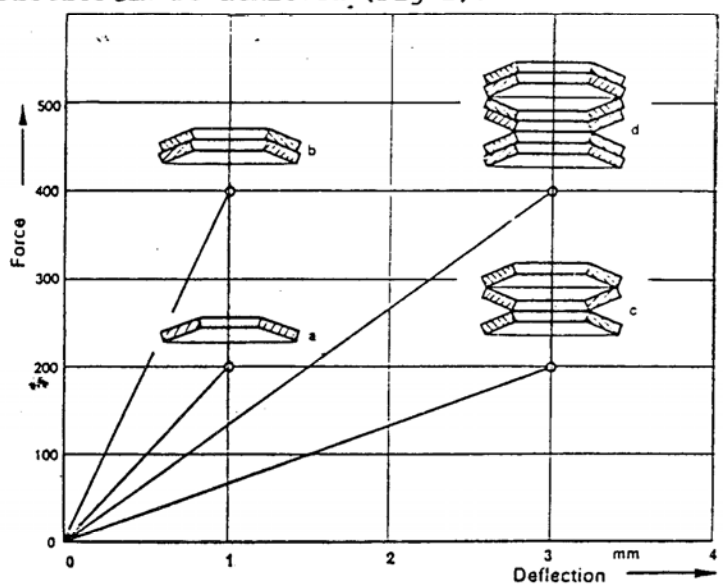


Fig 2 Force Deflection Characteristics for Various Spring Stacks

a) Single disc;
b) Two discs stacked in parallel (double force at same deflection);
c) Spring stack with three single springs stacked in series (triple deflection);
d) Spring stack with three sets in series of parallel pairs (double force and triple deflection).

Deflection and Load

There is a number of alternative methods for calculating the deflection and the stresses in disc springs but it is now accepted practice to use the elastic-disc method based on formulae developed by Almen and Laslo, provided that the ratio of the outside to inside diameter is less than 3.5. Thus, ignoring the frictional effects, the axial load P, for an axial deflection of δ, can be calculated from:-

$$P = \frac{4E\delta}{(1-\mu^2)D_o^2} \times \left[C_2(h-\delta).\left(h-\left(\frac{\delta}{2}\right)\right)t + C_2''t^3\right]$$

Where:-

D_o = Outside diameter

D_i = Inside diameter

μ = Poissons ratio
t = Thickness
E = Young's modulus
h = Cone height

$$C_2 = \pi\left(\frac{\alpha+1}{\alpha-1} - \frac{2}{\log_e\alpha}\right)\left(\frac{\alpha}{\alpha-1}\right)^2$$

$$C''_2 = \frac{\pi \log_e\alpha . \left(\frac{\alpha}{\alpha-1}\right)^2}{6}$$

Where

$$\alpha = \frac{D_o}{D_i}$$

It can be shown that, for all practical purposes, C_2 and C_2'' equal one another, so that the above formulae for load can be simplified to:

$$P = \frac{4E\delta C_2}{(1-\mu^2)D_o^2} \times \left[(h-\delta).\left(h-\left(\frac{\delta}{2}\right)\right)t + t^3\right]$$

To simplify the calculations of load still further, graphs are available which relate the factor C_2 as a function of the diameter ratio (α) and factor C_1 as a function of δ/t and h/t. Using the graphical approach, a simple load formula can be used, viz:-

$$P = \frac{4Et^4 C_1 C_2}{D_o^2} \quad \ldots\ldots\ldots\ldots\ldots (i)$$

The graph for establishing the load factor C_2 is given below:-

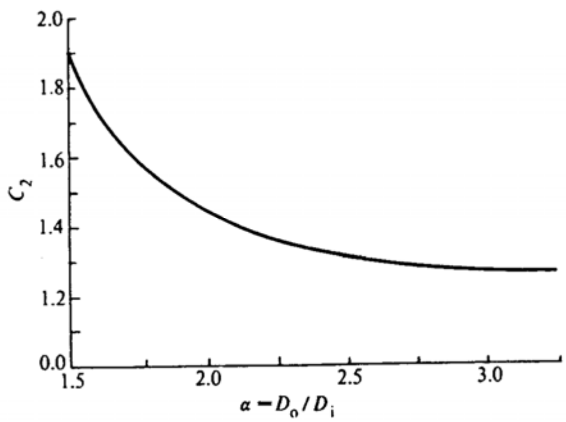


Fig 3

To establish the load factor C_1, according to the ratios for deflection and cone height to material thickness, the following curve is used:-

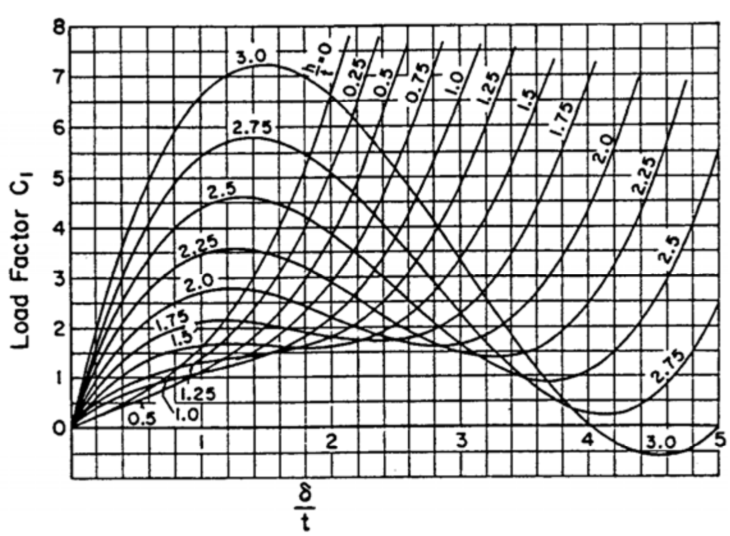


Fig 4

Stress

The formulae for calculating the stresses within a disc spring are based on elastic theory and are just as horrendous as those for determining deflection. Given below are the basic formulae for calculating the maximum stress at a point corresponding to the upper edge of the internal diameter of the disc - ie point A on the disc (compressive stress); the lower edge of the internal diameter - i.e. point B on the disc (tensile stress); and the lower outer edge of the outside diameter - i.e. point C on the disc (tensile stress).

$$f_A = -\frac{\delta t}{C_3\ D_o^2}\left[C_5 + C_4\left(\frac{h}{t} - \left(\frac{\delta}{2t}\right)\right)\right]$$

$$f_B = \frac{\delta t}{C_3 D_o^2}\left[C_5 - C_4\left(\frac{h}{t} - \frac{\delta}{2t}\right)\right]$$

$$f_C = \frac{\delta t}{C_3 \propto D_o^2}\left[C_5 + \left(2C_5 - C_4\right)\left(\frac{h}{t} - \frac{\delta}{2t}\right)\right]$$

Compressive stresses are indicated by a negative value whereas tensile stresses are shown as positive values.

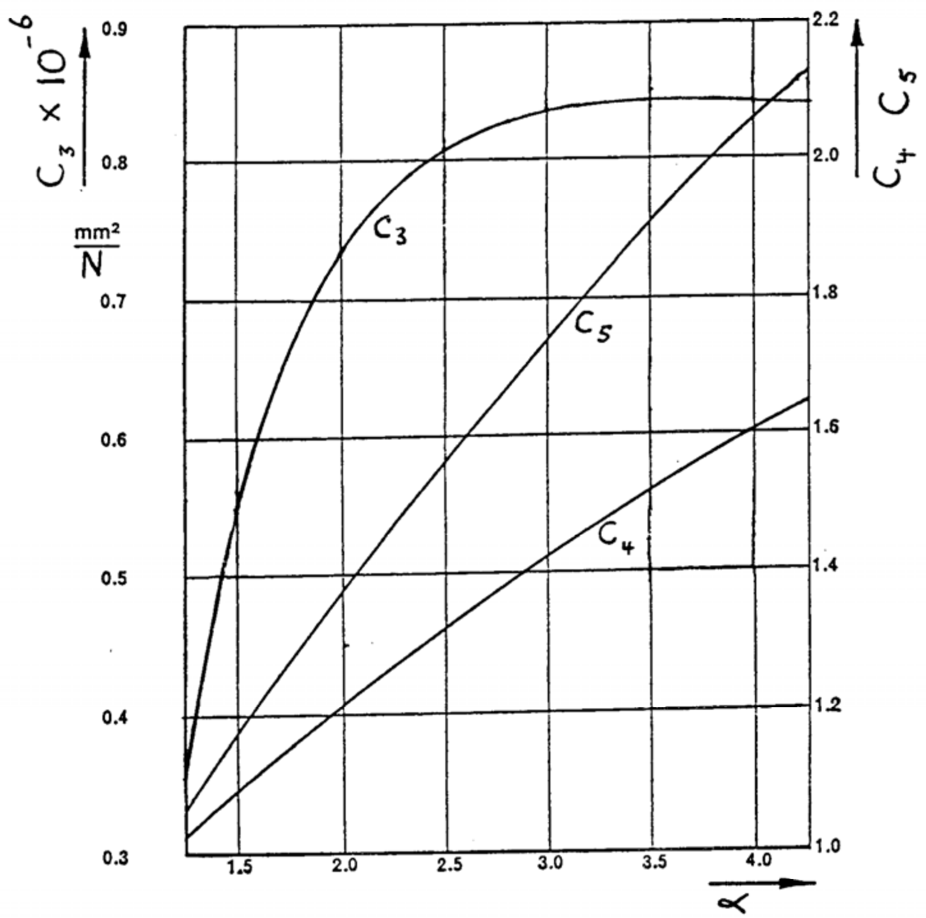
Where

$$C_3 = \frac{\left(\frac{\alpha - 1}{\alpha}\right)^2\left(1 - \mu^2\right)}{4E\pi\left(\frac{\alpha + 1}{\alpha - 1} - \frac{2}{\log_e\alpha}\right)}$$

$$C_4 = \frac{6}{\pi\log_e\alpha} \times \left[\left(\frac{\alpha - 1}{\log_e\alpha}\right) - 1\right]$$

$$C_5 = \frac{3\ (\ \alpha - 1)}{\pi\log_e\alpha}$$

A graphical solution to the stress factors C_3, C_4 and C_5 is given below in Fig 5 for a carbon steel (E = 208,000 N/mm^2 and = 0.3).



<u>Fig 5</u> Values of Coefficients C_3, C_4 and C_5

Disc springs are subject to compressive stress on the upper face and tensile stress on the bottom, the former being the major. For springs subject to a static or rarely-changing load, the limiting stress is therefore the compressive one at the upper face (f_A). However, for springs subjected to dynamic loads, the tensile stresses on the lower face (f_B and f_C) are the critical stresses and each must be calculated to determine which is the greater.

Design Example

A static load of 10,000 Newtons requires to be supported by a disc spring where the space available for the spring element is restricted to a diameter of 180 mm and a height of about 8.25 mm. In addition, the spring must be capable of supporting the specified load at a deflection of 7 mm with a possible 10% over run before it becomes solid and a maximum stress of 1450 N/mm^2.

i) Determine the deflection to available space:-

Required deflection = 7.0 + 10% = 7.7 mm

Maximum solid height = approx 8.25 mm

Ratio of deflection/solid height = $\frac{7.7}{8.25}$ = 0.93

ii) Select α, the Do/Di ratio:-

This is normally done by choosing a ratio somewhere in the middle of the range 1.5 to 3.5 - say $\alpha = 2.0$. If need be this can be modified at a later stage. To allow clearance on the 180 mm diameter space a disc having an outside diameter of 175 mm is selected which gives an inside diameter of Do = 87.5 mm.

iii) Determine the thickness:-

The thickness is determined from the maximum stress equation. The thickness must be such that the maximum stress is limited to an acceptable level. In this case the limiting value is 1450 N/mm^2.

As the spring application is static then the equation for the compressive stress at point A is the critical stress equation.

$$f_A = \frac{-\delta t}{C_3 \ Do^2}\left[C_5 + C_4\left(\frac{h}{t} - \left(\frac{\delta}{2t}\right)\right)\right]$$

To avoid instability in the load-deflection characteristics the h/t ratio must be less than 1.4. A convenient value of h/t = 1.0 is therefore selected.

The value of $\delta/t = 1.0$ represents the condition when a disc spring is compressed flat and the position at which maximum compressive stress occurs.

These values are now used in the stress equation to produce the following:=

$$f_A = - \frac{\delta t}{C_3 \; Do^2} \left[C_5 + C_4 \left(1 - \frac{1}{2} \right) \right]$$

As $\delta/t = 1$, then $\delta = t$

$$f_A = - \frac{t^2}{C_3 \; Do^2} \left[C_5 + C_4 \; (0.5) \right]$$

This formula can now be transposed to solve for t. The negative sign is now neglected as it was only an indication for type of stress (compression).

$$t = \sqrt{\frac{f_A \, C_3 \; Do^2}{C5 + C4 \; (0.5)}}$$

From Fig 5 the stress factors are determined as follows:-

$C_3 = .76 \times 10^{-6}$
$C_4 = 1.21$
$C_5 = 1.38$

Thus

$$t = \sqrt{\frac{1450 \times 0.76 \times 10^{-6} \times 175^2}{1.38 + 1.21 \; (0.5)}}$$

$t = 4.12$ mm

Since a reduction in thickness will reduce the stress, let us assume a convenient thickness of 4 mm.

The provisional design is now t = 4.00 mm, Do = 175, $\alpha = 2$, thus Di = 87.5 mm, h/t ratio = 1.0 thus cone height = 4.0 mm.

iv) Check on solid load and solid stress:-

$$P = \frac{4Et^4 C_1 C_2}{Do^2}$$

From Figs 3 and 4 determine C_2 and C_1 respectively.

C_2 for α of 2 = 1.45

C_1 for h/t of 1.0 and δ/t of 1.0 = 1.1

Thus solid load = $\frac{4 \times 208000 \times 4^4 \times 1.1 \times 1.45}{175^2}$

= 11093 N

v) Number of discs:-

Since the required deflection is for 7 mm at a 10,000 N load and the above spring only deflects 4 mm to solid then more than one spring will have to be stacked in series.

Determine load at deflection of 3.5 mm.

δ /t = .875 and h/t = 1
α = 2

Thus:-

$C_1 = 1$
$C_2 = 1.45$

$$P = \frac{4Et^4 C_1 C_2}{Do^2}$$

$$= \frac{4 \times 208000 \times 4^4 \times 1 \times 1.45}{175^2}$$

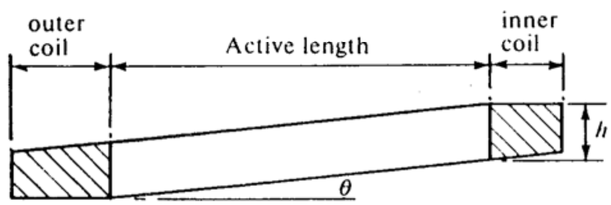
= 10080 N

Hence two discs stacked in series will double the deflection under the same load and give 7 mm deflection for 10080 N load.

VOLUTE SPRINGS

"Springs - materials, design and manufacture" defines a volute spring as "A spring produced from flat section material, helically coiled with its thickness in the radial direction such that each coil nests within its adjacent larger coil".

To achieve the required volute form it is necessary to fabricate the flat strip or flat bar stock to the developed shape shown in the diagram below prior to coiling.

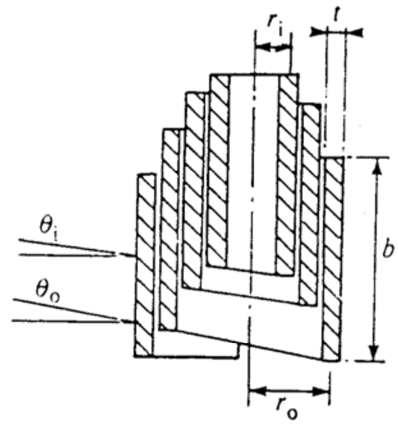


Developed blade for conventional volute spring

Normally there will be some friction between the coils which helps in damping vibration. However, for fatigue applications, the fretting of adjacent coils could lead to stress concentrations and premature failure. This can be avoided by designing the spring with space between the coils. Volute springs can be coiled with either a constant or variable helix angle and the rate characteristics will therefore depend on the nature of the helix angle variation. Springs coiled with a constant helix angle will possess both linear and non-linear portions of the load-deflection curve, the change representing increase in stiffness as the larger outer coils "bottom" and become inactive. This feature is often exploited in shock absorbers and railway buffers and also, due to the high volume efficiency of this type of spring, used extensively in suspensions for tanks, transporters and rolling stock. Volute springs can also be made from light gauge material and probably the most common domestic example would be the "secateur" spring used in garden pruning shears, which in fact is a double volute spring.

<u>Method</u> The mathematical approach to the design of volute springs is exceedingly complex and one solution to this problem is to use design charts and 'paired' variables. Although this causes some slight loss of accuracy the design effort is reduced considerably and the errors introduced are similar in magnitude to the manufacturing tolerances which must be allowed for such a spring.

Nomenclature and Design Chart



where:-

b	=	height of section
t	=	thickness of section
r_o	=	radius of largest active coil
r_i	=	radius of smallest active coil
δ_t	=	deflection from free to solid
W	=	load to 0.5 total deflection
σ_{max}	=	maximum stress
θ_o	=	helix angle of largest active coil (radians)
θ_i	=	helix angle of smallest active coil (radians)
n	=	number of free coils
N	=	total number of coils
G	=	rigidity modulus

Pairings:-

$$A = r_i/r_o \quad \text{........ } \underline{1}$$

$$B = \frac{1 - \theta_i/\theta_o}{1 - r_i/r_o} \quad \text{........ } \underline{2}$$

Y = Factor for deflection as a function of A and B

$$Z = 0.1 - 0.09A \text{ (load factor)} \quad \text{........ } \underline{3}$$

Formulae:-

$$W = \frac{YZGbt^3\theta_i}{r_o^2} \quad \text{........ } \underline{4}$$

$$\sigma_{max} = \frac{Gt\theta_i}{Ar_o} \quad \text{........ } \underline{5}$$

$$\delta_t = nr_o\theta_i Y \quad \text{........ } \underline{6}$$

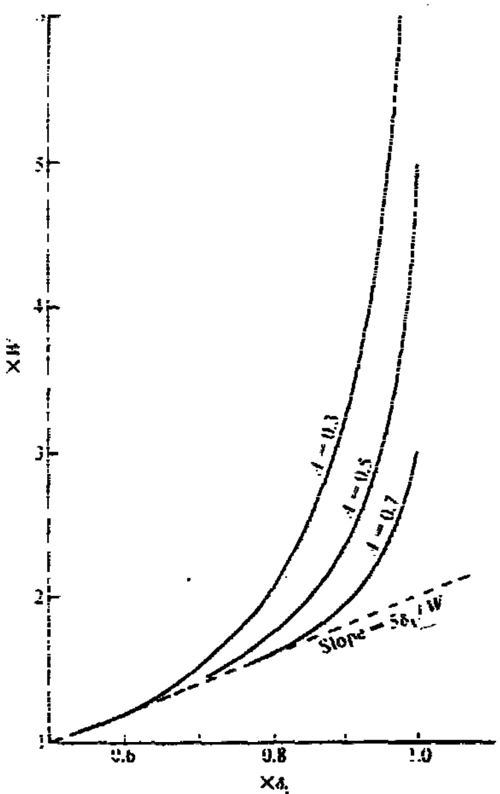


Fig. 1 Load plotted against deflection for $B = 0$

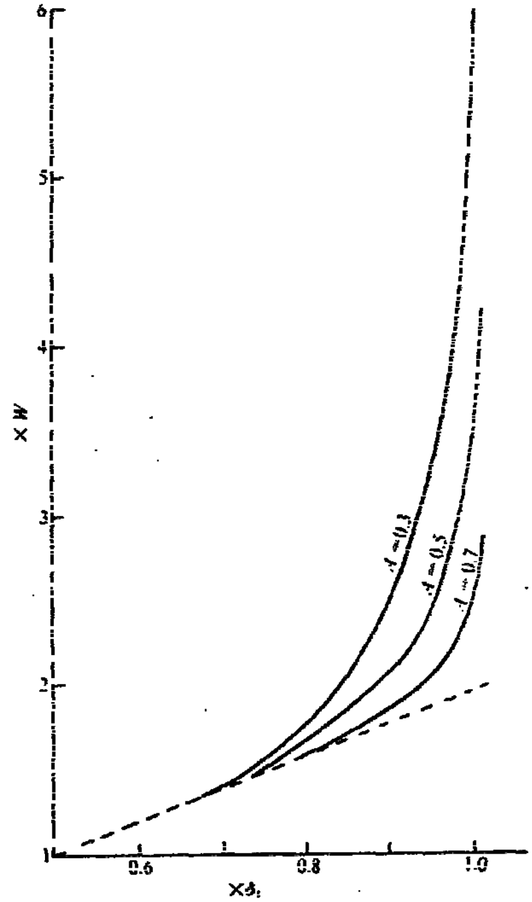


Fig. 2 Load plotted against deflection for $B = \frac{1}{2}$

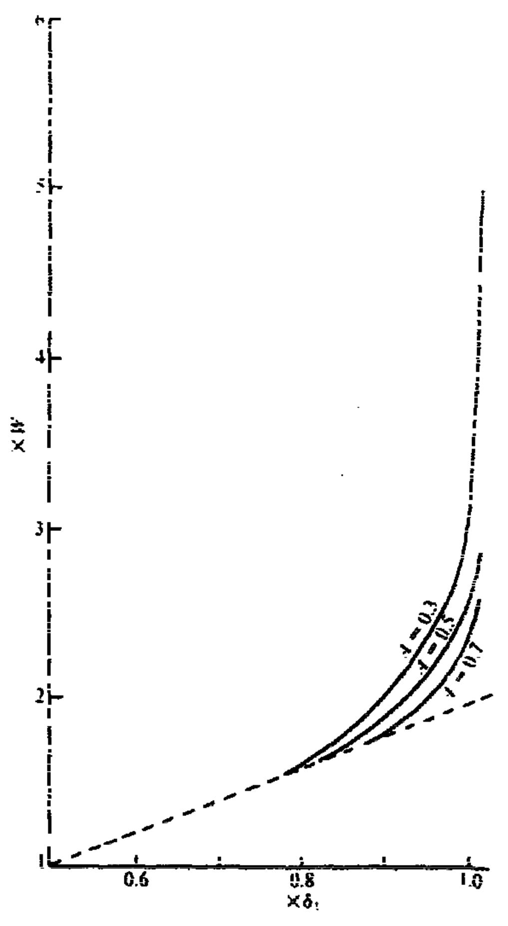


Fig. 3 Load plotted against deflection for $B = 1$

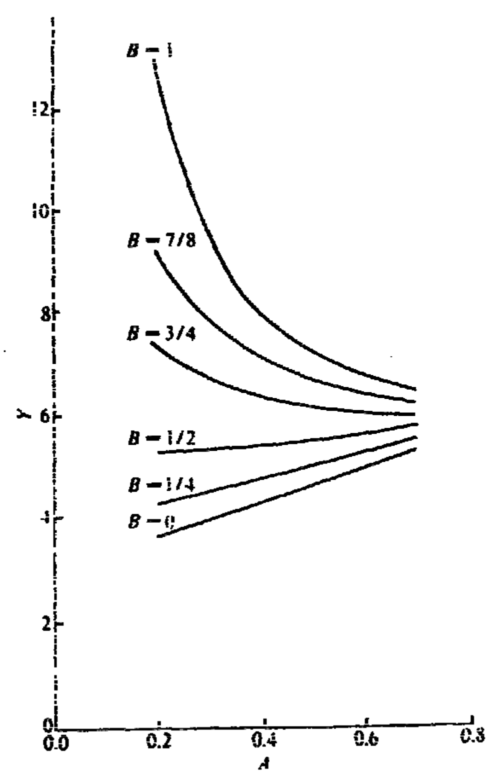


Fig. 4 Auxiliary factor Y

<u>Design Check</u>

Given all the pertinent information describing a volute spring it is a simple matter to check out the maximum stress, total deflection and loads at half and full deflection as follows.

Information given:- $b = 190mm$, $t = 10mm$, $N = 5.1/2$, $n = 4$, $r_o = 95mm$, $r_i = 50mm$, $\theta_o = 0.076$, $\theta_i = 0.060$, $G = 79000N/mm^2$.

i Calculate the 'pairings' A, B, Y and Z.

$$A = r_i/r_o = \frac{50}{95} = 0.526$$

$$B = \frac{1 - \theta_i/\theta_o}{1 - r_i/r_o} = \frac{1 - 0.789}{1 - 0.526} = \frac{0.211}{0.474} = 0.445$$

Y = 5.6 (from Fig 4.)

$$Z = 0.1 - 0.09A = 0.1 - 0.09 \times 0.526$$
$$= 0.1 - 0.04734$$
$$= 0.053$$

ii Calculate total deflection to solid from:-

$$\delta t = n r_o \theta_i Y$$
$$= 4 \times 95 \times 0.06 \times 5.6$$
$$= 128mm$$

iii Determine maximum stress from:-

$$\sigma_{max} = \frac{Gt\theta_i}{Ar_o}$$
$$= \frac{79000 \times 10 \times 0.06}{0.526 \times 95}$$
$$= 948.5\ N/mm^2$$

iv Determine load at 0.5 of total deflection

$$W = \frac{YZGbt^3\theta_i}{r_o^2}$$
$$= \frac{5.6 \times 0.053 \times 79000 \times 190 \times 10^3 \times 0.06}{95^2}$$

$$= 29618\ N.$$

v Load at solid length

From Fig 2, the multiplying factor is 3.25
Therefore solid load = 29618 x 3.25
= 96260 N.

Design Procedure

Using the design charts and 'paired' variables approach it is necessary, when designing volute springs from scratch, to make one or two dimensional estimates in order to work out a preliminary design which can subsequently be modified to meet the specified requirements. In many cases, however, a number of parameters such as load, deflection, maximum stress, solid height and outside diameter are specified. A simple design procedure is as follows:-

i Select a typical helix angle θ_i of 0.06 radians, (it may be necessary to adjust this figure at a later stage);

ii Guided by any information provided on the available space for the spring, estimate the radius of the largest active coil (r_o), but bear in mind and allow for the size of the dead-end coil;

iii Estimate r_i, typical ratios of r_i/r_o being between 0.3 and 0.7;

iv Calculate $A = r_i/r_o$;

v From information provided on the load requirements, choose a suitable load-deflection curve from Figs 1 to 3 and establish "pairing" factor B. Formula 2 can then be transposed and, using factor B, the helix angle of the largest active coil (θ_o) can be calculated;

vi Using Fig 4 the auxiliary deflection factor Y can be read off;

vii Calculate $Z = 0.1 - 0.09A$;

viii Assume a maximum stress (σ_{max}) at solid of 1000 N/mm^2 and, by transposing equation No 5, calculate the material thickness (t);

ix Since the load (W) at 50% of total deflection from free to solid should always be defined, the required width of section (b) can be calculated by transposing equation No. 4;

x The solid height of the spring is defined by the width of the section (b).

Example

A volute spring having a free length of about 200 mm is required to fit inside a hole of 200mm diameter. The load-deflection requirements of the spring are 9000N at a deflection of 40mm, increasing to 27000N at a deflection of 80mm, at which point the spring becomes solid. The maximum stress on the spring should not exceed $1000N/mm^2$:

i Select helix angle θ_i as 0.06 radians.

ii Estimate a value of r_o. Since the hole is 200mm the radius will be somewhat less than 100mm. Making an allowance for the thickness of the dead-end coil and for some clearance, 80mm would appear a reasonable estimate.

iii An estimate of the smaller end can then be made, as a guide line something around half the large end, say r_i = 30mm.

iv $A = \frac{30}{80} = 0.375$

v The ratio of the two load requirements is 27000/9000 = 3. and from Fig 2. an estimate of B = 0.5 can be made.

vi Using B = 0.5 and equation No. 2, a calculation of can be made:

$$0.5 = \frac{1 - (0.06/\theta_o)}{1 - 0.375}$$

$$\theta_o = 0.087$$

vii Reference to Fig 4 allows Y to be established as 5.5

viii Z can be calculated as 0.1 - 0.09 x 0.375
= 0.066

ix From equation No. 5 the thickness can then be calculated:

$$1000 = \frac{79000t \times 0.06}{0.375 \times 80}$$

$$t = 6.33\text{mm}$$

x Since the free height and required deflection are given, the solid height can be calculated as 200-80 = 120mm. This value fixes the maximum width of the material as b = 120mm.

xi The number of active coils can now be calculated using equation No. 6:

$$80 = n \times 80 \times 0.06 \times 5.5$$
$$n = 3.03$$

xii At this stage it is advisable to check the load at 0.5 solid using equation No. 4:

$$W = \frac{5.5 \times 0.066 \times 79000 \times 120 \times 6.33^2 \times 0.06}{80^2}$$

$$= 8182\text{N}$$

xiii Checking the stress (equation 5):

$$\delta_{max} = \frac{79000 \times 6.33 \times 0.06}{0.375 \times 80}$$

$$= 1000 \text{ N/mm}^2$$

xiv This initial design indicates a volute spring having the following dimensions:- b = 120mm, t = 6.33mm, n = 3, N = 4½ a large outside diameter of about 179mm (2 x 80 + 3 x 6.33) and a small inside diameter of about 41mm (2 x 30 - 3 x 6.33), the free height being 200mm.

xv Examination of xii above indicates the load at half the total deflection to be weak by about 800 Newtons and, from this point on, it is a question of making minor adjustments to the dimensions in order to meet the specification requirements. For example, increasing the thickness from 6.33mm to 6.5mm will increase this load to 8860N, only 40 newtons below the specified value. At the same time, of course, the max. stress will increase, in this case, by a negligible 26N/mm^2.

xvi Should it be necessary, further modifications to the dimensions of the spring can be made to meet more closely the specified requirements.

SPIRAL SPRINGS

Under this general heading are included springs coiled from flat strip in the form of a two dimensional spiral having space between the coils - open coil (brush springs or hair springs) - or tightly wound with contact between coils (power springs, motor springs or clock springs).

BRUSH /HAIR SPRINGS

These springs can be conveniently divided according to the method of end fixation, shape ratio (b/t) and number of coils, viz:-

1. Springs having a large number of coils and large shape ratio (e.g. 10):

    a. outer end of spring clamped rigid
    b. outer end located by a pin which allows rotation

2. Springs having few coils ($<2.1/2$) and a shape ratio less than 10.

Nomenclature:-

b = strip width
t = strip thickness
L = active length of strip
M = moment
n = number of revolutions (i.e. each of 360^0) produced by moment M
f = bending stress
E = Young's modulus

Case 1a. - Clamped outer end with many turns

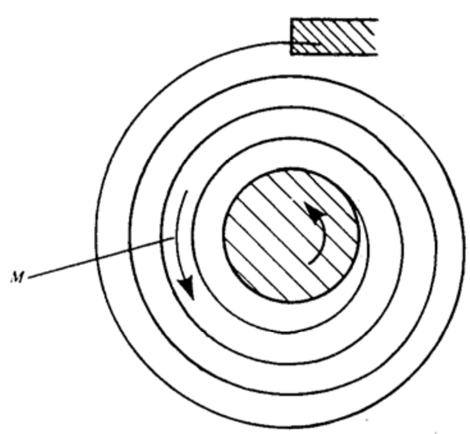


Fig 1. Open coil spring with clamped end and many coils

the standard formulae for springs of this type are:-

$$M = \frac{\pi E b t^3 n}{6L} \qquad \text{.......... (1)}$$

$$\text{therefore } n = \frac{6LM}{\pi E b t^3} \qquad \text{.......... (2)}$$

$$\text{and } f = \frac{6M}{bt^2} \qquad \text{.......... (3)}$$

Should it be necessary to crank either end of the strip for the purpose of fixing the material to the arbor or to the outer location point the operating stress in the vicinity of the bend could be substantially increased and a stress correction factor should be applied. This factor is dependent on the severity of the bend D/t, where D is twice the mean radius of curvature of the bend. The correction factor to apply in such cases is shown graphically in Fig 2 below:-

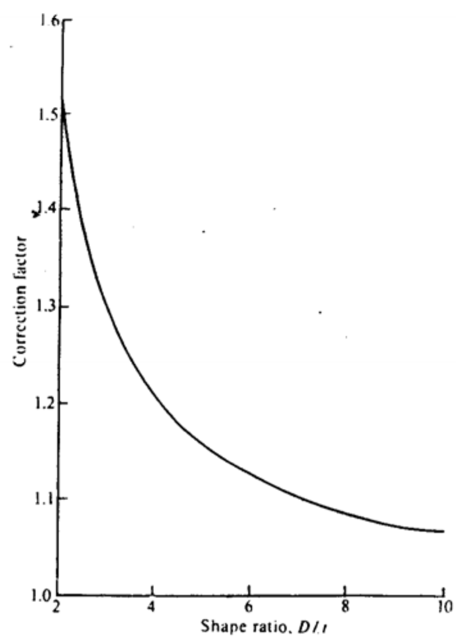


Fig 2 Correction factor for sharp bends

<u>Case 1b. - Pinned outer with many turns</u>

If the outer end of the strip is fitted over a pin which allows the end of the strip to swivel at this point then the Moment formula (1) given for clamped ends no longer applies. The modified formula for the moment becomes:-

$$M = \frac{2\pi E t^3 b n}{15L} \qquad \text{.......... (4)}$$

$$\text{therefore } n = \frac{15LM}{2\pi E t^3 b} \qquad \text{.......... (5)}$$

It will be seen that, for the same applied torque, a pinned outer end open coil spiral spring will have a lower rate than that for a clamped end spring. The stress also differs from that for a clamped end spring and for the same applied torque is in fact doubled:-

$$f = \frac{12M}{bt^2} \qquad \text{.......... (6)}$$

However, it should be noted that this maximum stress occurs at a point opposite the pinned end where there is no stress concentration.

Case 2. - Clamped outer end with few turns

Springs fitting this category include balance springs where the outer end is clamped and only allowed to move in an arc about the spring centre (see Fig 3.)

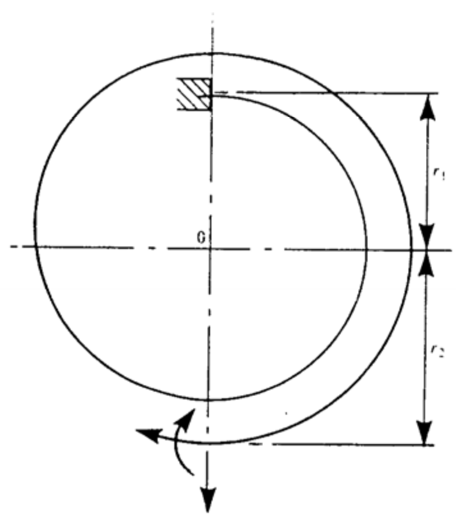


Fig 3. Spring with clamped ends and few coils

In cases such as this a modification to the standard stress formula (Equation 3) is necessary, using a factor which takes into account the spring shape (λ) and the number of turns (Θ) in degrees. The shape factor is defined as:-

$$\lambda = 1 - (r_1/r_2)$$

where r_1 is the inner end radius

r_2 is the outer end radius

The stress correction factor can best be illustrated graphically as in Fig 4.

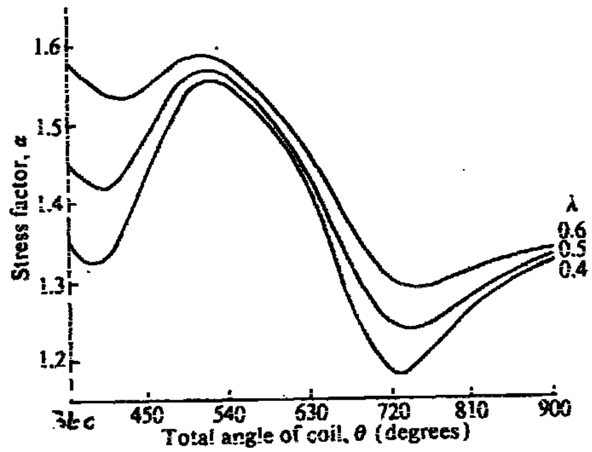


Fig 4. Stress factor

Likewise the rate of the spring needs modification by a factor which is related to spring shape (λ) and the total number of turns. This relationship is shown in Fig 5. below:-

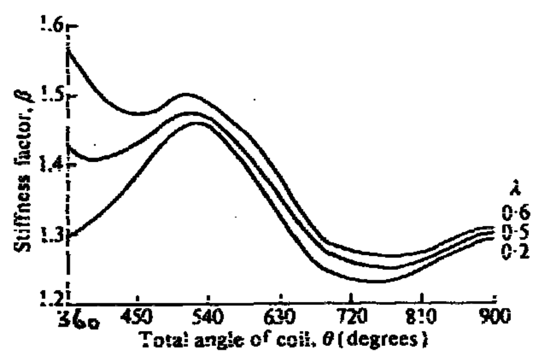


Fig 5. Stress factor β

Typical design stresses which are used for balance springs are given in the table below.

Material	Maximum Stress N/mm^2
Carbon	550
Stainless	520
Beryllium copper	420
Phosphor bronze	400

POWER SPRINGS

Power or clock springs consist of a spiral of flat strip anchored at its inner end to a central arbor, and at its outer end to the inner surface of a restraining band or case (see Fig 1). Usually the arbor attachment is accomplished by cranking the end of the strip and fitting it into a slot cut in the arbor. The outer end of the strip is often fixed via a hole in the strip which engages with a pin in the case. Since the strip materials used for power springs are invariably pre-hardened and tempered, it is necessary to soften the ends of the strip to facilitate hole punching and bending of the ends. Considerable care is needed during this localized annealing to restrict the length of softened material and to ensure a gradual transition from the soft to the hard zone.

Because of the inter-coil friction, the torque developed by power springs is somewhat erratic as the spring unwinds. Power springs invariably operate in the unwinding mode and, starting from the fully wound position, the delivered torque drops off rather slowly for about 50% of its travel. Further unwinding causes an increasing rate of torque decay.

Design Procedure

Practical experience has formulated guidelines to help the designer in the selection of some of the basic parameters:-

i To avoid excessively high stresses in the strip, the arbor diameter should be at least twenty times the strip thickness;

ii In general, the length of strip in the spring should not exceed 15,000 times the strip thickness if excessive inter-coil friction is to be avoided;

iii To obtain the best fatigue performance the edge of the strip should be rounded and preferably 'dressed'.

Nomenclature

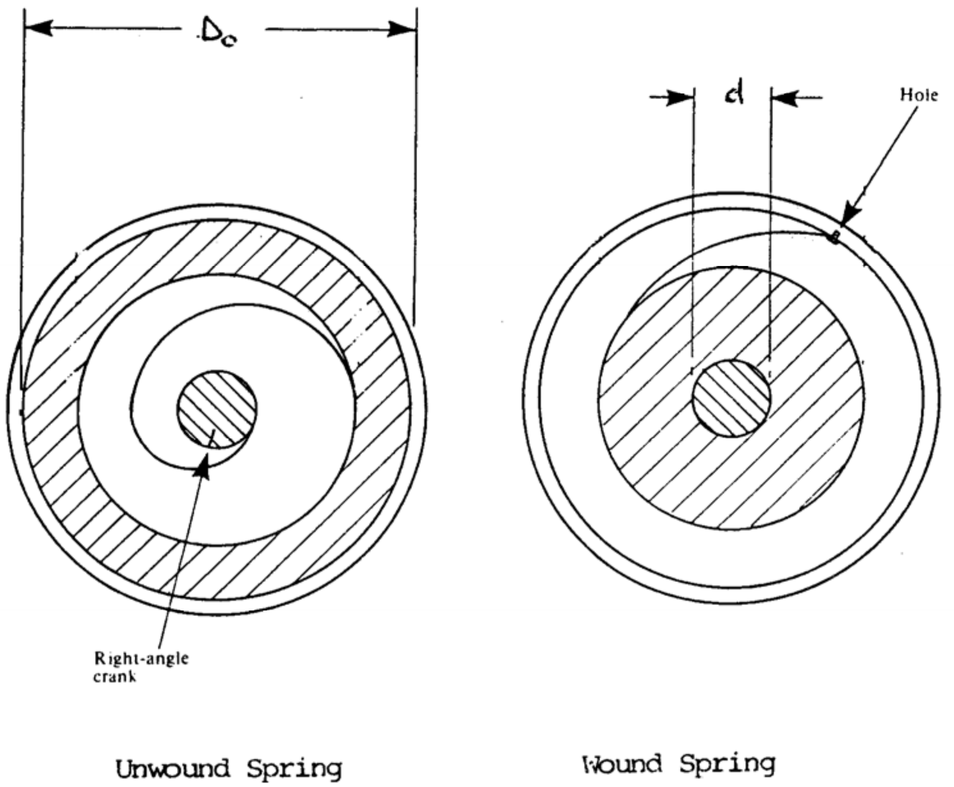


Fig 1 Power Spring in wound and unwound states

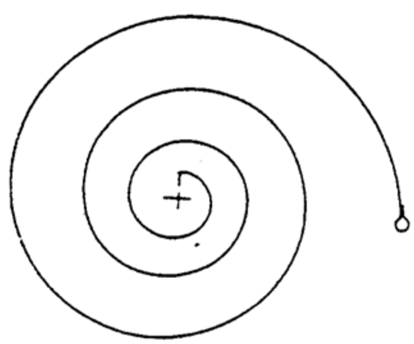


Fig 2 Configuration of power spring in free state (ie uncaged) showing 3¼ coils.

t = strip thickness
b = strip width
L = active length of strip
M = moment or torque at any position
M_{max} = maximum moment or torque when fully wound

f = bending stress at any position
f_{max} = maximum stress when fully wound

E = Young's modulus

D_o = inside diameter of the case

d = arbor diameter
n_o = number of coils in free (uncaged) state

n_c = number of active coils in unwound spring in case

n_s = number of active coils in fully wound spring in case

Δn = number of active coils delivered at any position
ΔN = total number of coils available

The basic formulae for the design of power springs can be given as:-

$$M = \frac{2\pi E b t^3 \left[(n_c - n_o) + \Delta n\right]}{12L} \qquad \text{........ (1)}$$

where
$$n_c = \frac{D_o - \sqrt{D_o^2 - \frac{4Lt}{\pi}}}{2t} \qquad \text{........ (2)}$$

$$n_o = \frac{(d + 2n_s t)\, n_s}{2D_o} \qquad \text{........ (3)}$$

and
$$n_s = \frac{\sqrt{\frac{4Lt}{\pi} + d^2} - d}{2t} \qquad \text{........ (4)}$$

From the above formulae it should be noted that the magnitude of the torque delivered (M) is dependent on the number of coils in the spring in the free state (i.e. uncaged). This is best illustrated by reference to the graph shown in Fig 3

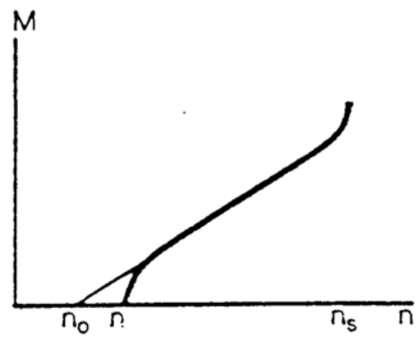


Fig 3 Torque curve for power spring

Having calculated the moment (torque) at any number of delivered turns (Δn) the corresponding bending stress can be calculated from:-

$$f = \frac{6M}{bt^2} \qquad \text{........ (5)}$$

The total number of available turns from a power spring is:-

$$\Delta N = n_s - n_c \qquad \text{........ (6)}$$

and therefore the maximum moment (torque) the spring will deliver is given by:-

$$M_{max} = \frac{2\pi Ebt^3 \left[(n_c - n_o) + \Delta N\right]}{12L} \qquad \text{........ (7)}$$

Likewise:-

$$f_{max} = \frac{6M_{max}}{bt^2} \qquad \text{........ (8)}$$

Because of the need to allow for an estimate of the number of coils in the free uncaged state and the problem of friction, the above formulae relating moment to the number of active turns can show inaccuracies over the initial and final portions of the load-deflection curve. This is shown in Fig 4 where the first 30% of wind-up and the last 10% of wind-up indicate substantial deviations from the torque calculated from equation (7):-

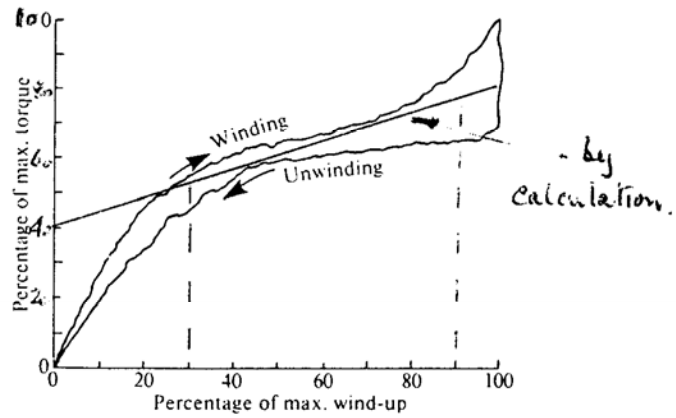


Fig 4. Typical torque - revolution curve for power spring

Where the maximum number of turns is required from a particular design, it has been shown in practice that the area occupied by the strip in the fully wound-up condition will be about 50% of the space available between the arbor and case. This optimum condition can be satisfied by choosing a combination of strip thickness and length such that

$$Lt = \frac{\pi\ (D_o^2 - d^2)}{8} \qquad \text{........(9)}$$

Worked Example

A power spring is required to give a maximum torque of 5000N mm from about 18 to 20 revolutions of an arbor of 20mm diameter. The space available for the spring allows a case with an inside diameter of 145mm and a depth which will accommodate a strip width of 28mm. The maximum bending stress should be limited to about 1700 N/mm^2.

Therefore data given is:- M_{max}= 5000 N mm

f_{max}= 1700 N/mm^2

D_o = 145 mm

d = 20 mm

b = 28 mm

i Calculate the thickness of strip from:-

$$t = \sqrt{\frac{6M_{max}}{bf_{max}}}$$

$$t = \sqrt{\frac{6 \times 5000}{28 \times 1700}}$$

$t = \quad 0.79$ mm

ii Determine active length of strip to maximise the total number of turns available (ΔN) from:-

$$L = \frac{\pi(D_o^2 - d^2)}{8t}$$

$$L = \frac{\pi(145^2 - 20^2)}{8 \times 0.79}$$

$L = \quad 10252$ mm

iii Calculate n_c n_s and n_o from equations (2), (4) and (3) respectively,

$$n_c = \frac{145 - \sqrt{145^2 - \frac{4 \times 10252 \times 0.79}{3.1416}}}{2 \times 0.79}$$

$= \quad 26.26$ coils

$$n_s = \frac{\sqrt{\frac{4 \times 10252 \times 0.79}{3.1416} + 20^2} - 20}{2 \times 0.79}$$

$= \quad 52.85$ coils

$$\therefore n_o = \frac{[20 + (2 \times 52.85 \times 0.79)] \times 52.85}{2 \times 145}$$

$= \quad 18.86$ coils

iv The total number of coils available (ΔN) can be calculated from:-

$\Delta N = \quad n_s - n_c$

$= \quad 52.85 - 26.26$

$= \quad 26.59$ coils

v Check the calculated value of coils available (N) meets the stipulated requirements. As explained earlier (see Fig. 4), an allowance should be made to ensure that the spring has approximately 40% more coils (by calculation) than the application stipulates. In the example here this condition is satisfied.

vi Finally, a check is made on the moment (torque) developed at the specified working position(s) using the equation (1). viz:-

$$M_{max} = \frac{2 \times \pi \times 207000 \times 28 \times 0.79^3 \left[(26.26 - 18.86) + 26.59\right]}{12 \times 10252}$$

$$= \quad 4961 \text{ N mm}$$

In this case the calculated moment is close to the specified value and is therefore acceptable.

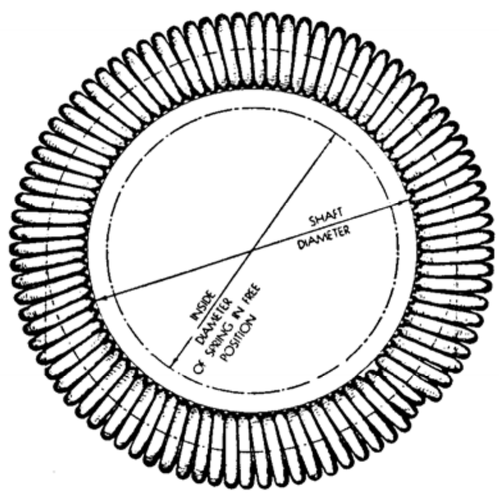
vii If the calculated maximum torque value does not meet the specified value it will be necessary, as a first option, to adjust the diameter of the case and re-calculate the torque. Remember that increasing the case diameter D_o will cause an increase in the effective length of the strip and also in the number of active coils, thereby reducing the maximum torque. Likewise, decreasing the case diameter will decrease the strip length and the number of coils which, in turn, will increase the delivered torque.

Should this optimising procedure fail to produce a spring within the guidelines given at the beginning of this article and/or the constraints imposed by the specification, it will be necessary to modify other dimensions such as the thickness and width of the strip.

Recommended maximum operating stresses for hardened and tempered high carbon steel vary according to the strip thickness from about 1100 N/mm^2 at 4mm to approaching 2000 N/mm^2 for very thin material around 0.25mm.

GARTER SPRINGS

Garter springs are long helical extension springs usually close coiled with initial tension. The two ends are joined together either by a 'connector' or by reducing one end for a short distance and screwing this into the unreduced end, to form a ring. The purpose of a garter spring is to exert radial forces to the object to which it is fitted and it finds a use in a variety of mechanical applications such as oil seals and small motor drive belts. The procedure for checking the design of garter springs is relatively simple but the initial design is complex and not normally carried out by springmakers. It is based on extension spring formulae with modifications to take into account the additional bending stresses which are imposed.



Nomenclature:-

d = wire diameter
D = mean diameter of the helix
D_{ri} = inside diameter of the garter spring
D_s = shaft diameter
P_o = initial tension
S = spring rate
c = D/d, spring index
K = correction factor = $(c + 0.2)/(c-1)$
n = number of working coils

E = Young's modulus
G = Rigidity modulus
p = radial force per unit length
q = combined stress due to bending and torsion
q_i = additional stress due to initial tension

Design Formulae

The basic formulae used for the purpose of checking the design involve the calculation of spring rate, the radial force exerted by the garter spring when fitted to a shaft or other similar object, and the total stress imposed on the spring due to torsion and bending, viz:-

$$D_{ri} = \frac{nd}{\pi} - \frac{d}{2}$$

$$S = \frac{dG}{8nc^3}$$

$$p = 2\left[\frac{P_o}{D_s} + \pi\left(1 - \frac{D_{ri}}{D_s}\right) S\right]$$

$$q = \left[\frac{\Delta D}{D} + \frac{2}{1 + 2G/E}\right] \cdot \frac{Gk}{nc}$$

where $\Delta D = D_s - D_{ri}$

$$q_i = \frac{8cP_oK}{\pi d^2}$$

Worked Example

A close coiled garter spring, having an initial tension of 9 Newtons and 170 active coils, is manufactured from 1.2mm carbon steel wire to produce a helix diameter of 6.8mm. When assembled, the garter is required to fit over a shaft of 76mm diameter. The radial pressure exerted on the shaft is unknown and needs to be determined, as does the working stress, to ensure the material is not overstressed, which would lead to plastic deformation of the spring and a loss in load.

d = 1.2mm
D = 6.8mm
c = D/d = 5.66
K = 1.258
n = 170

P_o = 9N
D_s = 76mm
G = 79.3kN/mm^2
E = 206.8kN/mm^2

i. Initially calculate the inside diameter of the garter spring when the ends are joined:-

$$D_{ri} = \frac{nd}{\pi} - \frac{d}{2}$$

$$D_{ri} = \frac{170 \times 1.2}{\pi} - \frac{1.2}{2}$$

$$D_{ri} = 64.4\text{mm}$$

ii. Calculate the increase in diameter when fitted on the shaft:-

$$\Delta D = D_s - D_{ri}$$

$$= 76-64.4$$

$$= 11.6\text{mm}$$

iii. The next step is to calculate the spring rate:-

$$S = \frac{dG}{8nc^3} = \frac{1.2 \times 79.3 \times 1000}{8 \times 70 \times 5.66^3}$$

$$= 0.386 \text{ N/mm}$$

iv. Using the calculated rate, the radial force per unit of circumference of the shaft can be determined:-

$$p = 2\left[\frac{P_o}{Ds} + \pi\left(1 - \frac{D_{ri}}{D_s}\right) S\right]$$

$$= 2\left[\frac{9}{76} + 3.1416\left(1 - \frac{64}{76}\right) \cdot 0.386\right]$$

$$= 0.62 \text{ N per mm of circumference}$$

v. The stress is now checked:-

$$q = \left[\frac{\Delta D}{D} + \frac{2}{1 + 2G/E}\right] \cdot \frac{Gk}{nc}$$

$$q = \left[\frac{12}{6.8} + \frac{2}{1 + \left(\frac{2 \times 79.3 \times 100}{20.6.8 \times 1000}\right)}\right] \cdot \frac{79.3 \times 1000 \times 1.258}{170 \times 5.66}$$

$$q = 300.2 \ \text{N/mm}^2$$

Note, for springs wound with initial tension, account should be taken of the additional stress developed as a result of the initial tension, viz:-

$$qi = \frac{8cP_oK}{\pi d^2} = \frac{8 \times 5.66 \times 9 \times 1.258}{3.1416 \times 1.2^2}$$

$$= 113.3 \ \text{N/mm}^2$$

Therefore the total stress $(q + q_i)$ = 300.2 + 113.3

= 413.5 N/mm^2